

# DCU School of Mathematical Sciences

## BASIC SKILLS WORKSHEET 1

### Calculations and calculators

*The aim of this worksheet is to revise numerical calculations and to ensure that we know how to use a scientific calculator properly.*

#### **What is a scientific calculator?**

This is a calculator that has exponential ( $e^x$ ), logarithmic ( $\log x$  or  $\ln x$ ) and trigonometric ( $\sin x$ ,  $\cos x$ ,  $\tan x$ ) functions on it as well as the basic operations ( $+$ ,  $-$ ,  $\times$ ,  $\div$ ). It must also have a memory feature (look for keys like  $M$ ,  $M^+$ ,  $M^-$ ), brackets (keys with  $($  and  $)$ , or maybe  $[$  and  $]$ ) and statistics functions (look for keys with  $\bar{x}$ ,  $\Sigma x$ ,  $Sx$ ). The bad news is that the calculator on your mobile phone is *not* a scientific calculator. Some scientific calculators have graph drawing tools on them. You will not need this facility.

#### **What is it for?**

It's for making our lives easier. No explanation necessary really. The ability to do long calculations in just seconds with a tool that costs just a few Euro has transformed the world. (When Einstein worked in Princeton University, he had to ring up an assistant who did his long sums for him.)

#### **The Golden Rule**

*A calculator will do what you tell it to do, but not necessarily what you want it to do.* That is, every calculator has certain rules that it obeys. You have to understand these rules in order to make it do what you want it to do.

#### **The basic operations: priorities.**

The basic operations are  $+$ ,  $-$ ,  $\times$ ,  $\div$ . Press these keys:

$$4, +, 7, \times, 3, =$$

(This means "press 4, then press  $+$ , then press 7,..." The commas are just to separate the terms.) The answer you get should be 25. That means the calculator understood what you typed as

$$4 + (7 \times 3)$$

rather than

$$(4 + 7) \times 3.$$

This is one of the built-in rules of your calculator. It has a priority list for the order of doing calculations which goes as follows:

1. expressions in brackets
2. functions (e.g.  $\sin x$ ,  $\sqrt{x}$ )
3. powers
4. multiplication and division
5. addition and subtraction

So in the example above, since multiplication has a higher priority than addition, the calculator did  $7 \times 3 = 21$  before adding on 4 to get the answer of 25.

### **Rounding off**

Calculate  $\frac{25}{17}$ . Your answer should look something like

$$1.470588235$$

In most circumstances, we would only be interested in the first few decimal places. For example, if this number were a price in Euros, we would only be interested in its value to the nearest cent. This means we want the number correct to two decimal places. To round off to two decimal places, we follow this rule:

- Check the value of the third decimal place.
- If this value is 4 or less, leave the first two decimal places as they are.
- If the value is 5 or more, add 1 to the second decimal place.

This rule works for rounding off to any number of decimal places. For example, to round off to 17 decimal places:

- Check the value of the 18th decimal place.
- If this value is 4 or less, leave the first 17 decimal places as they are.
- If the value is 5 or more, add 1 to the 17th decimal place.

So for example,

$$\begin{aligned} 1.230846 &= 1.2308 && \text{correct to 4 decimal places} \\ 1.230846 &= 1.231 && \text{correct to 3 decimal places} \\ 1.230846 &= 1.23 && \text{correct to 2 decimal places.} \end{aligned}$$

$$\begin{aligned} 52.62964 &= 52.6296 && \text{correct to 4 decimal places} \\ 52.62964 &= 52.630 && \text{correct to 3 decimal places} \end{aligned}$$

In the second line of the second example, the 1 added to the 9 gives 10, which then affects the previous decimal place.

## Brackets

These are a hugely useful tool on your calculator. It gives you a way of telling your calculator to do a certain part of a calculation before other parts. For example to calculate  $3 \times (4 + 7)$  you should type in

$$3, \times, (, 4, +, 7, ), =$$

Using its priority list, your calculator will automatically do the sum in brackets first, and then do the multiplication. What would the answer be if you omitted the brackets?

To calculate  $\frac{3+6}{5-2}$ , you need to tell the calculator to first of all add 6 to 3; then subtract 2 from 5; then divide the first answer by the second. So you should type

$$(, 3, +, 6, ), \div, (, 5, -, 2, ), =$$

It is important to learn how to do calculations like this “all in one go” rather than writing down the result of intermediate parts of the calculation: it helps with the accuracy of your answer. For example, suppose we wanted to know the value of  $\frac{2.67 + 1.62}{3.21 - 1.94}$  correct to one decimal place. Let’s do the calculation bit by bit:

$$\begin{aligned} 2.67 + 1.62 &= 4.29 \\ &= 4.3 && \text{correct to one decimal place.} \\ 3.21 - 1.94 &= 1.27 \\ &= 1.3 && \text{correct to one decimal place.} \end{aligned}$$

Now divide to get the answer:

$$\begin{aligned} \text{answer} &= \frac{4.3}{1.3} \\ &= 3.307692308 && \text{(according to my calculator)} \\ &= 3.3 && \text{correct to one decimal place.} \end{aligned}$$

Now do the calculation all in one go by typing in

$$(, 2.67, +, 1.62, ), \div, (, 3.21, -, 1.94, ), =$$

This gives

$$\begin{aligned} \frac{2.67 + 1.62}{3.21 - 1.94} &= 3.377952756 && \text{(according to my calculator)} \\ &= 3.4 && \text{correct to one decimal place.} \end{aligned}$$

So we see that *not* doing the sum all in one go can lead to errors.

## Function keys

These are built-in functions that the calculator knows how to evaluate, for example  $x^2$ ,  $\ln x$ ,  $\sin x$ , etc. To calculate  $13^2$ , press

$$13, x^2, =$$

To calculate  $\ln 1.05$ , press

$$\ln x, 1.05 =$$

To calculate  $\sqrt{4}$ , press

$$\sqrt{\phantom{x}}, 4, =$$

So for some functions, the number comes first and then the function key, while for others, it works to other way round. Try out all the keys on your calculator to see which way round they work. The order generally follows to order in which you would write down the function.

## The shift/2nd F key/INV

Your calculator should have a key with either “shift”, “2<sup>nd</sup> F” or *INV* or similar on it. This is used to access the second set of functions on the key pad. On my calculator, the second function on the  $x^2$  function key is  $x^{-1} = \frac{1}{x}$ . So for example, to calculate  $\frac{1}{3.2} = 3.2^{-1}$ , type in

$$3.2, 2\text{nd F}, x^2, =$$

The answer is 0.3125

## The mode keys

There are usually two of these keys. The main MODE key switches from the regular mode for doing calculations to the STAT mode for doing statistics. There should also be a key marked DRG. This allows you to switch between the different ways of measuring angles, D = degrees, R = radians, G = gradians. The units currently being used are displayed on the screen as DEG, RAD or GRAD.

## The $\pm$ key

This key, which might also be labelled “NEG” or “(-)”, allows you to enter a negative number. Typing “8,  $\pm$ ” enters the number  $-8$ . To calculate  $-6 \times (3 + 4)$ , type in

$$6, \pm, \times, (, 3, +, 4, ), =$$

## Some guidelines

These rules of thumb should be followed when using a calculator.

1. Check that you have pressed the correct keys.
2. Use brackets for ease and to get accurate answers.
3. Do a rough calculation in your head to see if your answer is in the right vicinity.

For example, suppose you calculated  $4.12 \times 5.31$  and got 218.772. Well, 4.12 is close to 4 and 5.31 is close to 5, so their product should be close to 20 rather than close to 200. So a quick check on your answer seems to indicate an error.

## Exercises

1. Carry out the following operations and explain how the calculator found the answer it gives.

(a) Press 4, +, 12, ×, 2

(b) Press 6, ÷, 2, −, 2

(c) Press 6, +, 7, ×, 4, −, 2

(d) Press 8, ×, 3, −, 6, ×, 2

2. For each of these calculations, write down a rough estimate of the answer and then check using your calculator.

(a)  $3.126 \times 5.603$

(b)  $(10.44)^2$

(c)  $\frac{40.302}{6.189}$

(d)  $\sqrt{30}$

3. Use your calculator to find the following numbers. Give your answers correct to 4 decimal places.

(a)  $\sqrt{36}$

(b)  $\sqrt{40.21}$

(c)  $\sqrt{0.01266}$

(d)  $2.34^2$

(e)  $0.402^2$

(f)  $\frac{1}{50.45}$

(g)  $\frac{34 + 106}{213 - 154}$

(h)  $\frac{1.403 - 2.116}{31.448 + 0.981}$

(i)  $\frac{\sqrt{44} + 5.3^2}{21.4 + 34.5}$

(j)  $\frac{1}{\sqrt{146.5}}$

(k)  $\frac{122 - \sqrt{401}}{\sqrt{344} + \sqrt{120}}$

(l)  $\sqrt{\frac{30.5}{12.6}}$

## Solutions to exercises

- Follow this priority list on p.2 to determine which part of the calculation the calculator does first.
  - $12 \times 2 = 24$  first, then add 4 to get 28.
  - $6 \div 2 = 3$  first, then subtract 2 to get 1.
  - $7 \times 4$  first, then add 6 and subtract 2 to get 32. Note that  $7 \times 4$  first, then subtract 2 and add 6 gives the same answer.
  - $8 \times 3 = 24$  and  $6 \times 2 = 12$  come first, then subtract 12 from 24 to get 12.
- To get a rough estimate, we do the calculation with the nearest whole numbers, or for nearby numbers which make the calculation easy to do. Answers are given correct to 4 decimal places.
  - Answer should be close to  $3 \times 6 = 18$ . (Answer = 17.5150)
  - Answer should be close to  $10^2 = 100$ . (Answer = 108.9936)
  - Answer should be near  $\frac{40}{6}$ , and this number is near  $\frac{42}{6} = 7$ . (Answer = 6.5119)
  - 30 lies between 25 and 36, and we know that  $\sqrt{25} = 5$ ,  $\sqrt{36} = 6$ . So the answer should be somewhere between 5 and 6. (Answer = 5.4772)
- If the answer has 4 or less decimal places on your calculator, there is no need to round off.
  - 6
  - 6.3411
  - 0.1125
  - 5.4756
  - 0.1616
  - 0.0198
  - 2.3729
  - 0.0220
  - 0.6212
  - 0.0826
  - 3.4566
  - 1.5558