DCU School of Mathematical Sciences

BASIC SKILLS WORKSHEET 4

Algebra II - Formulae and Transposition

The aim of this worksheet is to revise evaluating and transposing formulae.

Introduction

A major use of algebra is for writing down rules about e.g. science, economics, engineering in a concise way. For example, consider the total revenue generated by selling a certain number of cars at a certain price. We could write

$$\text{total revenue} = \text{number of cars} \times \text{price of each car}.$$

However it makes life a bit easier if we just abbreviate this by using some definitions. Let

$$\text{T} = \text{total revenue},$$
$$\text{P} = \text{price of car},$$
$$\text{Q} = \text{quantity (or number) of cars}.$$

Then we get the simple equation

$$T = PQ.$$

An equation like this which must always be true for the variables T, P, Q is called a formula.

Exercise 1

In the following examples, invent names (single letters) for the relevant quantities, and write down the formula which is described in words.

1. The average speed of a car is the distance travelled divided by the time taken.

2. The area of a rectangle is the length multiplied by the breadth.

3. Temperature in Farenheit is found by multiplying temperature in Celsius by 1.8 and then adding 32.

Substituting in formulae

- Making substitutions in formulae allows us to calculate the value of one quantity once the values of all the others are known. For example, the formula

  $$s = 4.9t^2$$

  gives the distance s (in meters) which an object falls in t seconds when it is dropped. So after $t = 1$ second, the object has fallen

  $$s = 4.9(1)^2 = 4.9\,\text{metres}.$$

  After $t = 4$ seconds, the object has fallen

  $$s = 4.9(4)^2 = 4.9(16) = 78.4\,\text{metres}.$$

- Consider the formula

  $$r = \sqrt{x^2 + y^2}.$$

  Evalaute r when $x = 5$ and $y = 12$.

  $$r = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13.$$

- Consider the formula

  $$f = 3x^2 - 4x + 6.$$

  We frequently use the notation

  $$f(x) = 3x^2 - 4x + 6$$

  to emphasise that the formula is telling us how the value of f depends upon the value of x. We might also write $T(P, Q) = PQ$ in the earlier example. If we are using this notation, then $f(2)$ means "the value of the formula when we substitute in $x = 2$". So for the example $f(x) = 3x^2 - 4x + 6$ we have

  $$\begin{aligned} f(2) &= 3(2^2) - 4(2) + 6 = 12 - 8 + 6 = 10, \\ f(1) &= 3(1^2) - 4(1) + 6 = 3 - 4 + 6 = 5, \\ f(-1) &= 3((-1)^2) - 4(-1) + 6 = 3 + 4 + 6 = 13, \\ f(-2) &= 3((-2^2)) - 4(-2) + 6 = 12 + 8 + 6 = 26. \end{aligned}$$

Exercise 2

1. Find the value of the unknown quantity in the given formula.

   (a) $F = 1.8C + 32$ Find F when $C = 36$.

   (b) $P = \dfrac{RT}{V}$ Find P when $R = 4$, $T = 275$ and $V = 22$.

   (c) $s = ut + \frac{1}{2}at^2$ Find s when $a = 9.8$, $u = 3.2$ and $t = 4$.

2. Given the formula $f(x) = -2x^2 + 3x + 6$, find

   (a) $f(1)$

   (b) $f(-1)$

   (c) $f(0)$

   (d) $f(2)$

   (e) $f(-2)$.

Manipulating formulae

The formula for converting temperature in celsius (C) to temperature in Farenheit (F) is

$$F = 1.8C + 32.$$

For example, this allows us to convert 15^o Celsius to 59^o Farenheit. But what if we wanted to convert from Farenheit to Celsius? Our present formula tells us *immediately* the value of F in terms of the value of C. We call it an explicit formula for F. We also say that F is the subject of the formula or that F is expressed in terms of C. What we want to do next is make C the subject of the formula, i.e. get C on its own on one side of the formula. We must manipulate the formula to do this. There is only one rule to follow:

Whatever we do to one side of the equation, we must also do to the other side.

The steps required to do this in the present case are as follows:

$$\begin{aligned} F &= 1.8C + 32 \qquad \text{start here} \\ F - 32 &= 1.8C \qquad \text{subtract 32 from both sides} \\ \frac{F-32}{1.8} &= C \qquad \text{divide both sides by 1.8.} \end{aligned}$$

This completes the process. We now have our Farenheit-to-Celsius formula where C is expressed in terms of F:

$$C = \frac{F-32}{1.8}.$$

Manipulating formulae - examples

The rule written down above is the only rule about manipulating formulae. After that, it is a matter of practice. There are some useful guidelines:

- If we want to make x the subject of a formula, try to get all the terms involving x together on one side, with anything *not* involving x on the other side.
- If x appears in the denominator of a fraction, multiply both sides of the formula by that denominator.
- If there is a square root in the formula, isolate it and then square both sides.

1. Make x the subject of the formula

   $$y = -4x - 12.$$

   Solution:

   $$\begin{aligned} y &= -4x - 12 \qquad \text{start here} \\ y + 12 &= -4x \qquad \text{add 12 to both sides} \\ \frac{y+12}{-4} &= x \qquad \text{divide both sides by -4} \end{aligned}$$

2. If $T = PQ$ express Q in terms of T and P.
Solution:

$$\begin{aligned} T &= PQ && \text{start here} \\ \frac{T}{P} &= Q && \text{divide both sides by } P \end{aligned}$$

There was only one step here. Dividing by P is just like dividing by any other number.

3. Make a the subject of the formula $v = u + at$.
Solution:

$$\begin{aligned} v &= u + at && \text{start here} \\ v - u &= at && \text{subtract } u \text{ from both sides} \\ \frac{v-u}{t} &= a && \text{divide both sides by } t \end{aligned}$$

4.

$$4x + 2z = yz + 8y$$

Express z in terms of x and y.
Solution:

$$\begin{aligned} 4x + 2z &= yz + 8y && \text{start here} \\ 4x + 2z - yz &= 8y && \text{subtract } yz \text{ from both sides} \\ 2z - yz &= 8y - 4x && \text{subtract } 4x \text{ from both sides} \\ z(2-y) &= 8y - 4x && \text{take } z \text{ out of both terms on the left-hand side} \\ z &= \frac{8y-4x}{2-y} && \text{divide both sides by } 2-y \end{aligned}$$

5.

$$y = \frac{2x+3}{x-1}$$

Express x in terms of y
Solution:

$$\begin{aligned} y &= \frac{2x+3}{x-1} && \text{start here} \\ y(x-1) &= 2x+3 && \text{multiply both sides by } x-1 \\ yx - y &= 2x+3 && \text{expand} \\ yx - y - 2x &= 3 && \text{subtract } 2x \text{ from both sides to get all } x\text{'s on the left} \\ yx - 2x &= 3 + y && \text{add } y \text{ to both sides to get everything not involving } x \text{ on the right} \\ x(y-2) &= 3 + y && \text{take } x \text{ out of both terms on the left-hand side} \\ x &= \frac{3+y}{y-2} && \text{divide both sides by } y-2 \text{ to finish.} \end{aligned}$$

6. Make b the subject of

$$a = 2\sqrt{\frac{c}{b}}.$$

Solution:

$$\begin{aligned}
a &= 2\sqrt{\frac{c}{b}} && \text{start here} \\
a^2 &= \left(2\sqrt{\frac{c}{b}}\right)^2 && \text{square both sides} \\
a^2 &= 2^2\left(\sqrt{\frac{c}{b}}\right)^2 \\
a^2 &= 4\frac{c}{b} \\
a^2 b &= 4c && \text{multiply both sides by } b \\
b &= \frac{4c}{a^2} && \text{divide both sides by } a^2 \text{ to finish.}
\end{aligned}$$

Exercise 3

1. Make Q the subject of the equation $P = -3Q + 35$.

2. $v = u + at$. Express t in terms of v, u and a.

3. $w = \dfrac{2v}{3}$. Express v in terms of w.

4. Make x the subject of the equation $y = \dfrac{2x+1}{x+4}$.

5. $a = \dfrac{3b-1}{2b+5}$. Express b in terms of a.

6. Make s the subject of the equation $t = 3\sqrt{\dfrac{s}{s+1}}$.

7. Make x the subject of the equation $v = \dfrac{k}{\sqrt{x}}$

8. Make x the subject of the formula $y^2 = 4 - x^2$.

9. $v^2 = u^2 + 2as$ Express u in terms of v, a and s.

Solutions to exercises

Exercise 1

1. Let s=average speed, d=distance travelled, t=time taken. Then $s = \frac{d}{t}$.
2. Let A=area, x=length, y=breadth. Then $A = xy$.
3. Let C= temperature in Celsius, F= temperature in Farenheit. Then $F = 1.8C + 32$.

Exercise 2

1. (a) 96.8
   (b) 50
   (c) 91.2
2. (a) 7
   (b) 1
   (c) 6
   (d) 4
   (e) -8

Exercise 3

1. $Q = \frac{35 - P}{3}$
2. $t = \frac{v - u}{a}$
3. $v = \frac{3w}{2}$
4. $x = \frac{1 - 4y}{y - 2}$
5. $b = \frac{1 + 5a}{3 - 2a}$
6. $s = \frac{t^2}{9 - t^2}$
7. $x = \frac{k^2}{v^2}$
8. $x = \sqrt{4 - y^2}$
9. $u = \sqrt{v^2 - 2as}$.