SM104
Beam Apparatus

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### SPECIFICATION

**Load Cells**
3 off, 0-40 N capacity with adjustable knife edges and locking screw for converting to rigid supports.

**Dial Gauges**
3 off, 0-25 mm in 0.1 divisions mounted on magnetic carriers.

**Spring Balance**
1 off, 0-10 N capacity.

**Weight Hangers**
4 off, fitted with cursors.

**Weights**
6 off, 10 N; 4 off, 5 N; 4 off, 2N.

**Cantilever Support**
1 off, rigid pillar with clamping arrangement suitable for all test beams.

**Beams**
3 steel, 1 brass, 1 aluminium, all 19 mm wide and 1350 mm long.

**Services Required**
NONE.

**Space Required**
Bench area approximately 2 x 0.6 m. Alternatively, the apparatus can span two small benches of the same height.

**Ancillaries**
The SM104a Specimen Beams, SM104b uniformly distributed loads, SM104c moving loads, additional load cells, dial gauges, hangers and/or weights can be supplied to order. Quote serial number of equipment when ordering.

### NOTE

**SPECIMENS**

The dimensions stated for specimens are for reference only. To ensure the accuracy of experimental results and calculations, it is recommended that all specimens are measured prior to experiments.
1. **INTRODUCTION**

The SM104 Mk III Beam Apparatus has many features which extend the range of experiments to cover virtually all coursework requirements relating to the bending of beams. The basic unit provides facilities for mounting beams on simple or sinking supports and also for clamping one end of the beam. Three load cells are supplied with the basic unit. These, together with the clamping unit, provide for up to four supports. Four hangers allow point loading at up to four points and three dial gauges allow simultaneous measurement of the deflections at three points. Traversing a dial gauge along the upper cross-member allows measurement of deflection along the whole length of the beam. Five beams are supplied as standard and all of these are 19 mm wide and 1350 mm long. Three of the beams are mild steel, one each of thicknesses 3 mm, 4.5 mm and 6 mm; a fourth beam is made of brass 6 mm thick and the fifth of aluminium alloy 6 mm thick. (Note: these are nominal dimensions).

The Beam Apparatus along with its ancillary equipment can be used for almost any experiment involving the investigation of beams subjected to bending under the action of point loads, distributed loads, moving loads and couples.

Additional equipment such as load cells, clamped supports, weights and hangers, dial gauges, special beams and loads are available from TecQuipment.
2. **UNPACKING AND ASSEMBLY**

2.1 **Unpacking**

1. Remove all of the components from their packing, taking care not to discard any packing material until the Packing Contents List has been checked.

2. Unwrap all of the components and lay them out on a suitable bench. Check the components against the Packing Contents List. Check amongst the packing material for any components not accounted for.

2.2 **Assembly**

The main frame of the SM104 is supplied dismantled in order to minimise packing and transport costs. To assemble the frame proceed as follows:

1. The cross-member with the graduated scale on it should be bolted to the tee legs through the upper holes (the brackets for the lower members face inwards). The scale should face forward and the cross-member should be bolted to the back of the upright members. The bolt head should face the front with a washer fitted below the bolt head. Use washers under all nuts and do not tighten the nuts until the whole frame has been assembled.

2. One of the other two cross-members has two brackets attached which will carry the beams not in use. This member is the rear of the two lower members and the brackets are at the back of the rear member. Bolt these two members to the tee legs through the lower holes, one each side of the legs. Again pass the bolts through from the front and, when all four nuts are in place, tighten up the six nuts.

3. The load cells are clamped to the lower cross-members with the upper guide behind the top cross-member and the cursor in front of the scale. The beam-clamping unit is clamped to the lower
members in the same way as the load cells. The load cells should be clamped by winding the plastic knob down and placing a 'C' washer above it. The plastic knob should now be wound up to clamp the load cell.

4. The dial gauges run on the top rail and are held in position by magnets. Before fitting the dial gauges remove the keepers from the magnets but do not discard them. Whenever the gauges are removed from the apparatus the keepers must be replaced.

5. The loading weights can be stored on rods mounted on the feet of the tee legs. These can be fixed either in front of or behind the apparatus, whichever is the more convenient.

6. The beam clamp can be attached at one end of the beam apparatus or stored until required.
3. **LOCKING OF LOAD CELLS**

The knife edges of the load cells can be locked in position by screwing the locating screw supplied through the DTI rest plate into the load cell body (as shown in Fig 0.1). Note the DTI should be removed before locating the knife edge.

**NOTE:** The load cells will be locked for shipment.
4. **FORCES DUE TO DIAL GAUGES**

In load-sensitive experiments it is necessary to include the load exerted by the dial gauges (Fig 0.2). This is best done by using calibration charts which may be constructed as follows:

Ensure the dial gauge is fitted properly to the top cross member or the load cell and the dial gauge contact is clear of obstructions. Zero the dial gauge. Ensure that the O-ring will allow the required movement.

Zero the spring balance with the crocodile clip attached. This is done by screwing or unscrewing the top of the balance unit until it is level with the zero marker. Attach the crocodile clip to the knurled part of the flat contact of the dial gauge (Fig 0.3).

Slowly raise the spring balance until a convenient deflection is indicated on the dial gauge (say 5 mm) and note the reading on the spring balance. Repeat this procedure, increasing the deflection each time.

A graph of force against deflection should be drawn. The graph should be linear.
Fig 0.2 Dial Gauge

Fig 0.3 Spring Balance Attachment

Fig 0.4 Calibration of Load Cells
5. **LOAD CELL CALIBRATION**

The three load cells can be calibrated by setting up a simply supported beam as shown in Fig 0.4. First set the beam horizontal with load cell A locked so that the support will not deflect when the beam is loaded; this done, set up a dial gauge on the beam above knife edge A and adjust to read zero. Now move the dial gauge to rest on the beam above the knife edge B and adjust the height of B so that the dial gauge again reads zero. Set the load cell at B to read zero. Load the beam at mid-span (do not forget to include the load due to the hanger, which can be found using the spring balance) and adjust the height of knife edge B using the thumb wheel until the dial gauge returns to zero. This is to ensure the beam is always horizontal and the load is perpendicular to the beam. Tap the frame gently and take the load cell reading. The force exerted on the load cell at B is W/2. Increase the load to some maximum value (e.g. 10 N) and then decrease to zero, taking the load cell reading at each step. Plot a calibration curve for the particular load cell B.

The procedure should then be repeated for the other load cells, taking care to identify each calibration curve with its own load cell. A typical curve is shown in Fig 0.5.

The load cells are calibrated so that 1 mm of downward displacement requires a force of 2 N. The number of revolutions of the dial gauge can be seen by the position of the rest plate relative to the indication marks below the main load cell body, as shown in Fig 0.6.
6. EXPERIMENTS

It is possible, especially if the additional equipment available from TecQuipment is obtained, to perform almost any experiment involving the bending of beams and cantilevers using the SM104 apparatus. The following experiments can be carried out using the basic unit and are described in detail in the following pages.

1. Support reactions for a point-loaded simply-supported beam
2. Variation of deflection of a simply supported beam with load, beam thickness and beam material
3. Verification of the theory of pure bending
4. Demonstration of reciprocal theory
5. Influence lines for deflection
6. Simulation of a rolling load
7. Continuous beam with fixed supports
8. Simply supported beam with central prop
9. Deflection of a cantilever
10. Propped cantilever with rigid prop
11. Propped cantilever with elastic prop

Experimental Note: As stated earlier, the dimensions of the specimens quoted in this manual are nominal. For accurate results the dimensions of the specimens should be measured by the student using suitable measuring instruments (not supplied).

6.1 Support Reactions for a Point-loaded, Simply-supported Beam

6.1.1 Introduction

This is a simple but useful experiment which allows students to familiarise themselves with the apparatus and its sensitivity and accuracy. The apparatus is set up as shown diagrammatically in Fig 1.1 with the beam supported at the 1/4-span points.

Before starting the experiment measure the length of the beam and mark it at mid-span and at the 1/4-span points for easy reference. Measure the thickness and width of the beam.
6.1.2 **Equipment Required**

Two load cells, one dial gauge, two load hangers, the weights and one of the beams.

6.1.3 **Procedure**

1. Choose a suitable reading on the upper scale of the apparatus for the mid-span of the beam. (One of the 10 cm markers is most convenient).

2. Set up one of the load cells so that it is 1/4-span to the left of the marker chosen in step 1. (Do not forget to take account of any offset in the position cursor).

3. Set up the second load cell 1/4-span to the right of the mid-span reading. Lock the knife edge.

4. Place the beam in position with 1/4-span overhang at either end.

5. Position two hangers equidistant from the mid point of the beam. (The cursors may press lightly against the scale).
6. Place a dial gauge in position on the upper cross member so that the ball end rests on the centre-line of the beam immediately above the left-hand support. Check that the stem is vertical and the bottom O-ring has been moved down the stem. Adjust the dial gauge to read zero and then lock the bezel in position. Move the dial gauge to a position above the right-hand support, check that the beam is parallel to the cross member, then adjust the height of the knife edge so that the dial gauge reads zero.

7. Remove the dial gauge and unlock both knife edges. Adjust the load cell indicators to read zero.

8. Apply loads to the hangers in a systematic manner, tap the beam very gently and take readings of the load cells.

9. Process the results and plot graphs to illustrate the theory.

6.1.4 Theory

Refer to Fig 1.2. Resolving vertically;

\[ R_1 + R_2 = W_1 + W_2 \]

Therefore: \[ R_2 = W_1 + W_2 - R_1 \] .... 1.1

Taking moments about \( R_2 \):

\[ R_1 \ell = W_1 \left( \frac{1}{2} \ell + a \right) + W_2 \left( \frac{1}{2} \ell - b \right) \]

\[ \frac{1}{2}(W_1 + W_2) + W_1a - W_2b \]

Therefore \[ R_1 = \frac{1}{2}(W_1 + W_2) + W_1a/\ell - W_2b/\ell \] .... 1.2

Substitute 1.2 in 1.1:

\[ R_2 = \frac{1}{2}(W_1 + W_2) - W_1a/\ell + W_2b/\ell \] .... 1.3
6.1.5 Analysis of Results

Beam size: \( L = 1350 \text{ mm long}, \ 19 \text{ mm wide}, \ 6 \text{ mm thick} \)
Beam working length: \( \ell = 675 \text{ mm} \)
\( a = b = 150 \text{ mm} \)
\( \Delta = (R_1 + R_2) - (W_1 + W_2) \)
\( \% = 100\Delta / (W_1 + W_2) \)

<table>
<thead>
<tr>
<th>( R_1 ) (N)</th>
<th>( W_1 ) (N)</th>
<th>( W_2 ) (N)</th>
<th>( R_2 ) (N)</th>
<th>( R_1 + R_2 ) (N)</th>
<th>( \Delta ) (N)</th>
<th>( % )</th>
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</table>
6.1.6 Comparison with Theory

Substituting the value of \( \lambda, a \) and \( b \) in equations 1.2 and 1.3, the following table is obtained:

<table>
<thead>
<tr>
<th>( W_1 ) (N)</th>
<th>( W_2 ) (N)</th>
<th>( R_1 ) (N)</th>
<th>( R_2 ) (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0</td>
<td>3.611</td>
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<td>0</td>
<td>14.444</td>
<td>5.555</td>
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<td>30</td>
<td>0</td>
<td>21.666</td>
<td>8.333</td>
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<td>5</td>
<td>1.388</td>
<td>3.611</td>
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<td>2.777</td>
<td>7.222</td>
</tr>
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<td>14.444</td>
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<td>30</td>
<td>30.000</td>
<td>30.000</td>
</tr>
</tbody>
</table>

Fig 1.2 Theoretical arrangement
6.2 Variation of Deflection of a Simply Supported Beam with Load, Beam Thickness and Material

6.2.1 Theory

The theory of pure bending of a beam can be found in any standard textbook and shows that when a beam is loaded in such a way that it bends only in the plane of the applied moment, the stress distribution and curvature of the beam are related by

\[ \frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R} \]  \( \ldots \ldots \) 2.1

where \( M \) is the bending moment
\( I \) is the second moment of area of the beam section
\( E \) is the Modulus of Elasticity
\( R \) is the radius of curvature
\( \sigma \) is the bending stress at distance \( y \) from the neutral axis
\( y \) is the distance from the neutral axis

It can also be shown that the curvature of a beam \( l/R \) is given, to a close approximation, by the second derivative of the deflection. If \( z \) is the deflection of the beam at distance \( x \) from a chosen origin then:

\[ \frac{d^2z}{dx^2} = \frac{1}{R} = \frac{M}{EI} \]  \( \ldots \ldots \) 2.2

Using equation 2.2 it can be shown that the deflection of a beam subjected to direct loading can always be expressed in the form:

\[ z = a \frac{Wx^3}{EI} \]  \( \ldots \ldots \) 2.3

where \( z \) is the deflection
\( a \) is a constant whose value depends upon the type of loading and supports
\( W \) is the load acting on the beam
L is the span
E and I are defined above

is the relationship investigated in this experiment.

2 Equipment Required

load cells, one dial gauge, one hanger, set of weights, all of the beams.

6.2.3 Procedure

1 - 4 As for experiment 1.
5. Place the hanger at mid span so that the loading point is on the centre-line of the beam.
6. Exactly as for experiment 1.
7. Place the dial gauge at mid span so that the ball end of the plunger rests on the centre of the setscrew. Adjust the dial to read zero and lock the bezel.
8. Apply a load to the hanger and record the beam deflection on the dial gauge. Note:- the scale dimensions on the dial gauge are 0.1 mm i.e. 10 m.
9. Increase the load and record the new dial gauge reading (deflection). Do this at least five times. Decrease the load by the same steps as 9 and record the beam deflection at each step.
11. Repeat the experiment for all of the beams.
12. For each beam plot a graph of deflection against load. Determine the gradient of each graph.
13. For the three steel beams plot a graph of the gradient obtained in step 12 against l/d² (d is the thickness of the beam).
14. Taking $E_S = 21 \times 10^{10}$ N/m², $E_B = 10.5 \times 10^{10}$ N/m², $E_A = 7.6 \times 10^6$ N/m² and using the values obtained for the 6 mm thick beams, plot a graph of the gradient obtained in step 12 against 1/E.
6.2.4 Analysis of Results

<table>
<thead>
<tr>
<th>LOAD W (N)</th>
<th>DEFLECTION z (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>STEEL 6 mm</td>
</tr>
<tr>
<td></td>
<td>STEEL 4.5mm</td>
</tr>
<tr>
<td></td>
<td>STEEL 3 mm</td>
</tr>
<tr>
<td></td>
<td>BRASS 6 mm</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>15</td>
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</table>

Table 1 Steel and Brass Beams

<table>
<thead>
<tr>
<th>Load W (N)</th>
<th>Deflection z (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AL 6mm</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
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<td>4</td>
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</table>

Table 2 Aluminium Beam

Fig 2.1 shows the typical shape of graphs obtained by plotting the results in tables 1 and 2.
6.2.5 Comments

The graphs of deflection vs load are straight lines for all five beams. The transverse stiffness of the beams can be determined from the slopes of the lines (stiffness = \( W + \text{[deflection in mm]} \times 10^{-3} \)).

For the five beams the stiffness obtained from fig 2.1 can be recorded as in the table below.

Theoretically the stiffness of a beam is proportional to \((\text{thickness})^3\), i.e. \(\text{stiffness}/(\text{thickness})^3 = \text{constant}\). Fig 2.2 shows that, for the three 6 mm beams, the slope of the lines of fig 2.1 are proportional to \(1/E\) in accordance with the theory. The slope of this line is proportional to \(L^3/I\).

<table>
<thead>
<tr>
<th>Material</th>
<th>thickness (mm)</th>
<th>Stiffness (N/m)</th>
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</thead>
<tbody>
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</tr>
<tr>
<td>Steel</td>
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<tr>
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<td>Brass</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Aluminium</td>
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<td></td>
</tr>
</tbody>
</table>

Fig 2.2 \( z/W \sim 1/E \)
6.3 Verification of the Theory of Pure Bending

6.3.1 Introduction

The theory for a beam subjected to bending was outlined at the start of Experiment 2 (section 6.2). The object of this experiment is to show that equation 2.2 is valid.

The radius of curvature is measured by the Sagital method. Referring to Fig 3.1, the radius of curvature is obtained from the relationship:

\[ h(2R - h) = a^2 \]

ie \( 2Rh - h^2 \approx a^2 \) in which \( h^2 \) is usually very small

Therefore \( \frac{1}{R} = \frac{2h}{a^2} \)

Also from Equation 2.2:

\[ \frac{1}{R} = \frac{M}{EI} \text{ and } M = Wb \]

Therefore \( h = \frac{Wa^2b}{2EI} \) .... 3.1

Fig 3.1  Fig 3.2

6.3.2 Equipment Required

Two load cells, three dial gauges, two hangers, set of weights, one of the beams preferably 6 mm thick.
6.3.3 Procedure

To set up the apparatus so that the beam is subjected to a constant bending moment over the middle half of its length follow steps 1 to 4 of experiment 1.

5. Place one hanger near the left-hand end of the beam so that the load is applied on the centre-line of the beam. Place the second hanger at the right-hand end of the beam so that the hangers are the same distance from the supports. (See fig 3.2).

6. Place one dial gauge at mid-span and the other two equidistant on either side of it as shown in fig 3.2.

7. Adjust the three dial gauges to read zero.

8. Apply equal loads to the two hangers.

9. Gently tap the beam near its mid-span and take the readings of the dial gauges, $h$, $h$, and $h$.

Increase the load and repeat steps 8 and 9 for at least five values of $W$.

11. Process the results as shown in the following example
6.3.4 **Analysis of Results**

Beam - Brass, 6 mm thick 1350 mm long

\[ y = \frac{1}{2}(h_1 + h_3) \]

<table>
<thead>
<tr>
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<th>( W )</th>
<th>( b ) (mm)</th>
<th>( a ) (mm)</th>
<th>( h_1 ) (mm)</th>
<th>( h_2 ) (mm)</th>
<th>( h_3 ) (mm)</th>
<th>( y ) (mm)</th>
<th>( h_2 - y ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Test 1</strong></td>
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</table>

These results can be presented in graphical form as shown in Figs 3.3 and 3.4. \( \frac{h_2 - y}{b} \) (mm)
6.3.5 Calculations

The straight line graphs obtained by plotting $h^2 - y$ (this is $h$ in equation 3.1) against $W$ show that equation 3.1 is of the correct form, since from this equation the gradient is:

$$\frac{a^2 b}{2EI}$$

Thus when $b$ is 300 mm ( = .3m) the gradient = $\frac{.15a^2}{EI}$

Therefore for constant $b$ (in this case 0.3 m) a graph of gradient against $a^2$ should be linear and its gradient will give a measure of the flexural rigidity of the beam.

The second moment of area of the beam $I = \frac{19 \times 6^3}{12 \times 10} = 3.42 \times 10^{-10} \text{ m}^4$

Using the above results:

$$EI = \frac{.15a^2}{\text{gradient}}$$

Therefore:

$$E = \frac{.15a^2}{\text{Grd. x I}} \text{ N/m}^2$$

6.4 Demonstration of Reciprocal Theory

6.4.1 Theory

Referring to fig 4.1a, it can be shown that the deflection at D of the simply supported beam AB with point load W at C is given by:

$$z_D = \frac{Wab (L^2 - a^2 - b^2)}{6L} \ldots \ldots 4.1$$

It is seen that the value of $z$ will not change if $a$ and $b$ are interchanged, i.e. if the loading is as shown in Fig 4.1b, then the deflection at F is the same as the deflection at D.
If the beam is now loaded as shown in Fig 4.1c then the deflection at G will be the same as the deflection at D. This is a particular case of reciprocal theory which is demonstrated in this experiment. The general theory should be referred to in a standard textbook.

Fig 4.1 Reciprocal Theory

6.4.2 Equipment Required

Two load cells, three dial gauges, three hangers, weights, one beam.
6.4.3 Procedure

1-4 Exactly as in Experiment 1.
5. Place load hangers and dial gauges as shown in fig 4.2. (Note: a and b need not be equal but are equal in the example below.)
6. Use one of the dial gauges to go through step 6 of experiment 1. Return the dial gauge to its original position.
7. Apply a suitable load to any hanger, record the readings of all three dial gauges.
8. Move the load to the other two hangers in turn and record the readings of all three dial gauges.
9. Various combinations of loads can be applied and inter-changed recording the readings of all three dial gauges.
10. Use the results to demonstrate the reciprocal theory as shown below.

Fig 4.2 Demonstration of Reciprocal Theory
6.4.4 Analysis of Results

\[ a = b = 200 \text{ mm} \]
\[ W = \text{Load (N)} \]
\[ h = \text{deflection (mm)} \]

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
 & W_1 & h_1 & W_2 & h_2 & W_3 & h_3 \\
\hline
i & 0 & 5 & 0 & 0 & & \\
ii & 5 & 0 & 0 & & & \\
iii & 0 & 0 & 5 & & & \\
iv & 0 & 10 & 0 & & & \\
\hline
\end{array}
\]

a) SINGLE POINT LOAD

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
 & W_1 & h_1 & W_2 & h_2 & W_3 & h_3 \\
\hline
i & 0 & 5 & 0 & 0 & & \\
ii & 10 & 5 & 0 & & & \\
iii & 0 & 15 & 0 & & & \\
iv & 0 & 0 & 10 & & & \\
\hline
\end{array}
\]

b) TWO POINT LOADS

6.4.5 Examination of Results

Comparison of results a(i), a(ii), and a(iii), will verify that reciprocal theory applies to points 1 and 2, to points 2 and 3 and also to points 1 and 3. Comparison of results a(i) and a(iv) will verify that the deflection is doubled when the applied load is doubled.

Examination of the part b results will verify the following:
a) From (i) and (ii) the additional deflections at 2 due to applying the additional 10 N at point 1 = \( x_1 \).

b) From (i) and (iii) the additional deflection at 1, due to applying the additional 10 N at point 2 = \( x_2 \).

These two values \( x_1 \) and \( x_2 \) should agree to within a very small percentage.

c) From results b(ii) and b(iv) it should be clear that reciprocal theory applies to points 1 and 3 and from results b(iii) and it will also be seen to apply to points 2 and 3.

6.5 Influence Line for Deflection

6.5.1 Introduction

The influence line for the deflection of a point on a beam is a line showing the relationship between the deflection of that point and the position of application of a unit load acting on the beam. The deflection of the point due to the application of any load to the beam is obtained by applying the principle of superposition after multiplying the ordinate of the influence line by the magnitude of the load. (See the example which follows).

6.5.2 Equipment Required

Two load cells, dial gauge, hanger, weights, one beam.

6.5.3 Procedure

1-4 As for experiment 1.

5. Set up the equipment as shown in fig 5.1 with the load cells locked.

6. Carry out the levelling operation as for experiment 1.

7. Vary the value of \( W \) for chosen values of \( a \) and \( b \) (fig 5.1) and record the dial gauge readings.
8. For each set of readings determine the mean deflection per increment (see the results).

9. Plot the influence line for 'unit' load. The 'unit' may be defined as the incremental load.

10. Apply a load system and compare the measured deflection with that given by calculation based on superposition and the influence line.

Fig 5.1 Schematic of Experimental Set-up

Fig 5.2 Influence lines for \( W = 5 \) & \( W = 10 \) N
6.5.4 Analysis of Results

\[ a = 200 \text{ mm} \quad h = \text{deflection (mm)} \]
\[ k = \text{deflection /5 N (mm)} \]

\[ (h \text{ at } W=5 - h \text{ at } W=0)/5 + (h \text{ at } W=10 - h \text{ at } W=5)/5 + \ldots \text{etc} \]

<table>
<thead>
<tr>
<th>b (mm)</th>
<th>450</th>
<th>400</th>
<th>350</th>
<th>300</th>
<th>250</th>
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<tr>
<td>W (N)</td>
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These results can be presented in graphical form as in fig 5.2.

6.5.5 Example of Application of the Influence Line for \( W = 5 \)

1. \( 10 \) N applied at \( b = 450 \) mm together with \( 5 \) N at \( b = 250 \) mm
   
   Deflection due to \( 10 \) N = \( 2 \times k \) at \( 450 \) (mm)
   
   Deflection due to \( 5 \) N = \( k \) at \( 250 \) (mm)
   
   Total Deflection = \( 2 \times k \) at \( 450 \) + \( k \) at \( 250 \) (mm)

   The measured deflection is then compared with the total deflection obtained above. It should be found that the two figures are in close agreement.

2. \( 5 \) N applied at \( b = 400 \) mm together with \( 10 \) N at \( b = 200 \) mm
   
   Deflection due to \( 5 \) N = \( k \) at \( 400 \) (mm)
   
   Deflection due to \( 10 \) N = \( 2 \times k \) at \( 200 \) (mm)
   
   Total Deflection = \( k \) at \( 400 \) + \( 2 \times k \) at \( 200 \) (mm)

   Again the measured deflection should be in very close agreement with the total deflection.
6.6 Continuous Beam with Fixed Supports

6.6.1 Introduction

A great amount of care is required if good results are to be obtained from this experiment. Attention must be paid to accurate adjustment of the heights of the knife edges and correction of the load cells for backlash. Small errors in the reading of the load cells can lead to difficulty in interpreting the results.

It can be shown from the theory for a continuous beam carried on rigid supports, as shown in fig 6.1, that:

\[
\frac{M_2}{W} = \frac{3(a^2 + b^2)}{16(a + b)} \quad \ldots \ldots 6.1
\]

\[
\frac{R_1}{W} = 0.5 - \frac{M_2}{W} \quad \ldots \ldots 6.2
\]

\[
\frac{R_2}{W} = 0.5 - \frac{M_2}{W_b} \quad \ldots \ldots 6.3
\]

\[
\frac{R_3}{W} = 2 - R_1 - R_2 \quad \ldots \ldots 6.4
\]

![Fig 6.1 Continuous Beam on 3 Supports](image-url)
6.6.2 Equipment Required

Three load cells, three dial gauges, two hangers, weights, one beam.

3 Procedure

1. Set up the load cells at convenient positions so that they will support the beam near its ends and at 1/3 span.

2. Place the beam in position with equal overhang at either end.

3. Place a hanger at the middle of each part of the span.

4. Set the load cell pointers to zero

5. Place a dial gauge in position above R1. If the gauge has no O-ring on the upper part of the stem it requires one fitting. Move the O-ring down the stem so that it allows contact just to be made with the beam. Set the zero, lock the bezel, do NOT move the O-ring.

6. Move the dial gauge from R1 to R2 and adjust the height of the knife edge so that the dial gauge just contacts the beam, no further.

7. Repeat 6 with the dial gauge at R3.

8. Repeat 5, 6, and 7 until the beam is horizontal, making sure that the load cells indicate zero.

9. Place O-rings on the upper part of the stems of the remaining dial gauges then place gauges at R1, R2, and R3, so that the ball just contacts the beam. (There should be sufficient pressure to just displace the load cell indicator.) Set the zeros of the dial gauges.

Place equal loads on the hangers.
11. Adjust the height of the knife edges so that contact is just regained with the dial gauges then take the load cell readings.

12. Repeat 10 and 11 until sufficient results have been obtained

6.6.4 Analysis of Results

BEAM Steel 6 mm thick
ARRANGEMENT Refer to fig 7.1 a 400 mm b 800 mm

<table>
<thead>
<tr>
<th>W (N)</th>
<th>R₁</th>
<th>R₁/W</th>
<th>R₂</th>
<th>R₂/W</th>
<th>R₃</th>
<th>R₃/W</th>
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6.6.5 Calculated Values

Substituting the values of (a) and (b) in equations 6.1 to 6.4 the following results are obtained:

\[
R₁ = \frac{0.5 - \frac{125}{400}}{W} = 0.1875
\]

\[
R₂ = \frac{0.5 - \frac{125}{800}}{W} = 0.3438
\]

\[
R₃ = 2 - 0.1875 - 0.3438 = 1.4687
\]

Comparing the above values with those obtained by experiment it should be seen that there is close agreement.
Simply Supported Beam with Central Prop

1 Theory

The theory for beams supported by rigid or elastic props should be referred to in the standard textbook. For the configuration shown in Fig 7.1 it can be shown that the force induced in the rigid prop is given by:

\[ P = 1.375 W \]  \hspace{0.5cm} \ldots \ldots 7.1

It can also be shown that the deflection at mid-span is:

\[ \frac{11}{12} \times \frac{Wl^3}{EI} \]  \hspace{0.5cm} \ldots \ldots 7.2

Fig 7.1 Simply Supported Beam with Central Prop

6.7.2 Equipment Required

Three load cells, dial gauge, two hangers, weights, one beam.

6.7.3 Procedure

Carry out the usual setting up procedure but in this case it is not essential to support the beam at the 1/4 points.

5. Arrange the hangers as shown in Fig 7.1.

6. Lower the knife edge at mid-span so that it is clear of the beam. Lock the other two knife edges.

7. Place a dial gauge at mid-span and set it to read zero.
8. Set the pointer on the middle load cell to zero.
9. Apply loads to the two hangers.
10. Measure the deflection at mid-span. Make sure that the beam is not in contact with the knife edge. If necessary press the knife edge down with a finger whilst the deflection is measured.
11. Adjust the knife edge by screwing up until the dial gauge returns to zero. Record the load cell reading.
12. Alter the load repeating 9 through 12 as many times as required.

6.7.4 Analysis of Results

Steel beam 6 mm thick; 19 mm wide; span 1200 mm; I = 342 (mm)^4.

<table>
<thead>
<tr>
<th>W (N)</th>
<th>Deflection z (mm)</th>
<th>P' (N)</th>
<th>P/W</th>
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6.7.5 Comments

1. P/W should agree very closely with equation 7.1.

2) From equation 7.2: \[ E = \frac{11 x WL^3}{384Iz} \]

Therefore: \[ E = \frac{11 \times 5 \times 1200^3}{384 \times 342 \times z} = 72.368 \times 10^6 \text{ N/mm}^2 \] where \( W = 5 \text{ N} \)
Deflection of a Cantilever

6.8.1 Introduction

Reference to the textbook will show that the deflection under the load for a cantilever loaded at the free end is given by:

\[ z = \frac{WL^3}{3EI} \]  \hspace{1cm} \ldots \ldots 8.1

If \( EI \) and \( L \) are maintained constant then:

\[ z = k_1W \]  \hspace{1cm} \ldots \ldots 8.2

where \( k_1 \) is constant

Similarly if \( EI \) and \( W \) are maintained constant:

\[ z = k_2L^3 \]  \hspace{1cm} \ldots \ldots 8.3

Likewise \( z = k_3 \), and \( z = k_4 \) if \( E \) and \( I \) respectively are made the variables.

It follows therefore that experiments can be carried out showing the relationship between the deflection and each of the quantities \( E, I, L \) and \( W \).

In the experiment illustrated below only \( L \) and \( W \) are varied.

Note Since the deflection of the cantilever due to self weight is large, the cantilever is made to deflect upwards by screwing the knife edge of a load cell upwards. For high accuracy the loading of the beam due to the dial gauge should be taken into account.

Equipment Required

Clamped support, one load cell, one dial gauge, one beam or more.

6.8.3 Procedure (Refer to Fig 8.1.)

1. Set up a load cell at a convenient position near to one side of the frame.
2. Set up the clamp to give a cantilever of convenient length.
3. Pass one end of the beam through the clamp and rest the other end on the load cell. (It is convenient to lock the knife edge during assembly). Tighten the clamp and tie up the free end of the beam using a short piece of string.

4. Place the dial gauge near to the clamp and set the zero. Move the dial gauge to the free end of the cantilever, unlock the knife edge and adjust it so that the dial gauge returns to zero. Set the pointer of the load cell to zero.

5. Adjust the knife edge upwards to give a convenient reading on the load cell. Record the load and the dial gauge reading.

6. Adjust the knife edge upwards to give a number of load increments recording loads and dial gauge readings.

7. Return the knife edge to its initial position; lock the knife edge; slacken the clamp and move it to a new position (this is more convenient than moving the load cell).

8. Repeat the experiment for several lengths of cantilever.

9. In order to vary E use the Aluminium, Brass and Steel beams 6 mm thick.

10. In order to vary I use the steel beams 3 mm, 4.5 mm and 6 mm thick.

Fig 8.1 Schematic of Experimental Set-up
8.2 Graphs
6.8.4 Typical Results

BEAM Steel 6 mm thick

<table>
<thead>
<tr>
<th>L (mm)</th>
<th>200</th>
<th>300</th>
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<th>600</th>
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<tbody>
<tr>
<td>W (N)</td>
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6.8.5 Comments

The graphs of $z$ vs $W$ verify that equation 8.2 is correct, with $k_1$ for each length being given by the gradient of the graph. The graph of $z$ vs $L^3$ verifies that equation 8.3 is correct, with $k_2$ for each load being given by the respective gradient of the graph.

If the gradients of the graphs for $z/W$ are obtained for each length and a graph of $x/W$ vs $L^3$ is plotted, the gradient of the straight line obtained will give a measure of the flexural rigidity of the beam when substituted in equation 8.1.

From 8.1:  

$$z = \frac{WL^3}{3EI}$$

$$\frac{3EI}{z} = \frac{WL^3}{z}$$

since $I = 342 \text{ mm}^4$ then:  

$$E = \frac{\text{inverse of gradient}}{3 \times 342} \text{ N/mm}^2$$
6.9 Propped Cantilever (with Rigid Prop)

6.9.1 Introduction

The theory for propped beams and cantilevers can be referred to in most standard textbooks.

For a cantilever loaded as shown in Fig 9.1 with a rigid prop at its free end it can be shown that:

a) the deflection at the free end, if the prop is absent, is given by:

\[ z = \frac{W}{2} \frac{L^2 - a^2}{6} \]  

... 9.1

b) for a rigid prop the force in the prop is: \[ P = \frac{P}{16} \]  

... 9.2

Since the deflection of a cantilever loaded at its free end is given by

\[ z = \frac{PL^3}{3EI} \]

then \( P, W \) and \( a \) are related by the equation:

\[ \frac{PL^3}{3EI} = \frac{Wl^3}{3EI} \left(3c^2 - c^2\right) \]  

... 10.3

where \( c = \frac{a}{L} \)

In this experiment the relationships given in equations 9.2 and 9.3 are investigated.

Fig 9.1 Loaded Cantilever Supported by Rigid Prop at End
6.9.2 **Equipment Required**

Load cell, Dial Gauge, Hanger, Weights, Clamp, 6 mm thick aluminium beam*, steel straight edge.

* A beam of other material may be used but the deflection of the free end due to self loading necessitates the use of the short cantilevers.

6.9.3 **Procedure** (Refer to Fig 9.2)

1. Set up a load cell at a convenient position so that a cantilever 1 m long can be accommodated. Lock the knife edge.

2. Locate the clamp at its approximate position. Remove the four allen screws and support the beam in the recess of the clamp so that one end rests on the knife edge. Ensure that the beam is parallel to the upper cross member of the frame then replace the top of the clamp and replace the four allen screws so that the beam is only lightly clamped. Now, with a rule held vertically in contact with the clamp, set the length of the cantilever as required and tightly clamp the beam and lock the clamp to the lower cross members. (See Fig 9.3.)

3. Place a dial gauge on the beam as close to the clamp as possible and set it to read zero. Lock the bezel.

**Fig 9.2 Experimental Set-up**
4. Remove the dial gauge and place it on the beam above the knife edge.

5. Unlock the knife edge and adjust its height so that the dial gauge reads zero. (Note: Be sure that the 0 - indicated by the pointer is the zero and not +100 or -100 divisions. If necessary, slide the dial gauge along the whole length of the cantilever and watch the pointer).

6. Place the dial gauge in position above the knife edge and set the load cell pointer to zero.

7. Place the hanger in position.

8. Adjust the height of the knife edge so that the dial gauge reads zero.

9. Record the load cell reading.

10. Add a load to the hanger and repeat steps 8, 9 and 10 so as to obtain at least five results.

11. Remove the weights and hanger and repeat step 6.

12. Place the hanger at 0.2 of the span (measured from the clamp) and apply a suitable load (e.g. 8 N).

13. Adjust the height of the knife edge so that the dial gauge reads zero and record the load cell reading.

14. Now place the load at 0.4 of the span and repeat for 0.5, 0.6 and 0.8.

15. Either plot graphs to verify equations 10.2 and 10.3 or arrange the results in tabular form as shown in the example below.

Fig 9.3
6.9.4 Analysis of Results

Beam Aluminium 6 mm thick
Hanger = 1.7 N
Part 1: L = 1000 mm a = 500 mm c = 0.5

Part 2: L = 1000 mm W = 9.7 N

Note: From equation 9.3: \( P/c^2 = \frac{1}{4}W(3 - c) \)
i.e. \( W = \frac{P/c^2}{(1.5 - 0.5c)} \)

<table>
<thead>
<tr>
<th>W</th>
<th>ΔW</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>ΔP</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>c</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P/c^2</td>
<td>1.4</td>
<td>1.35</td>
<td>1.3</td>
<td>1.25</td>
<td>1.2</td>
<td>1.1</td>
</tr>
<tr>
<td>1.5-0.5c</td>
<td>1.4</td>
<td>1.35</td>
<td>1.3</td>
<td>1.25</td>
<td>1.2</td>
<td>1.1</td>
</tr>
<tr>
<td>W</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error %</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6.9.5 Comments

The average values of \( \frac{ΔP}{ΔW} \) from part 1 agree with equation 9.2,

\( \text{i.e.} \frac{P}{W} = \frac{5}{16} \)

The results of Part 2 verify equation 9.3 and show that the greatest experimental errors occur when the load cell is lightly loaded. Notice also that the load cell reading for \( c = 0.2 \) are multiplied by a much larger number then the reading for \( c = 0.8 \); therefore small errors at \( c = 0.2 \) are highly magnified compared with the same errors at \( c = 0.8 \).
6.10 Propped Cantilever (With Elastic Prop)

6.10.1 Introduction

Referring to the introduction to experiment 9, equation 9.1 still applies in the present case and is restated here as the first of the equations applying to a cantilever loaded as shown in fig 10.1:

\[ a) \quad z_w = \frac{W}{EI} \frac{La^2}{2} - \frac{a^3}{6} \]

\[ b) \quad \text{If the prop force is } P \text{ then:} \]

\[ i) \quad \text{the deflection of the cantilever due to the prop force is:} \]

\[ z_p = \frac{P L^3}{3EI} \]

\[ ii) \quad \text{the deflection of the prop due to the prop force is:} \]

\[ \delta = \frac{P}{k} \]

where \( k \) is the stiffness of the prop.

It is obvious that the deflection of the prop must be equal to the difference between the deflections given by 10.1 and 10.2.

\[ \text{i.e.} \quad \delta = z_w - z_p \]

and if \( a = cL \) then:

\[ \frac{W L^3}{6EI} (3c^2 - c^3) \quad \frac{P L^3}{3EI} = \frac{P}{k} \]

\[ \text{whence } \frac{P}{W} = \frac{3c^2 - c^3}{2 + (6EI/kL^3)} = \text{constant} \]

\[ \ldots \ldots 10.4 \]
6.10.3 Equipment Required

Load cell, dial gauge, hanger, weights, clamp, 6 mm thick aluminium beam, steel straight-edge.

6.10.4 Procedure

1-9 Exactly the same as for experiment 9.
10. Add successive loads to the hanger recording the values of load, deflection and load cell reading.
11. Remove the load and hanger and repeat experiment 8 (see note).
12. Compare the value of the Modulus of Elasticity obtained in this experiment with that obtained in step 11.

6.10.5 Analysis of Results

The stiffness of the load cell (elastic prop) is easily determined (see page 3).

The experimental set-up was shown in Fig 10.2.

The results can be presented in tabular form as shown below.

<table>
<thead>
<tr>
<th>W (N)</th>
<th>ΔW (N)</th>
<th>z (mm)</th>
<th>Δz (mm)</th>
<th>P (N)</th>
<th>ΔP (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>5</td>
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<td>15</td>
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<td>35</td>
<td>5</td>
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</tr>
<tr>
<td>AVERAGE</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

6.10.6 Calculations

From equation 11.4 \[ P = \frac{(3 \times 0.5^3) - 0.5^3}{W (6EI/kl^3) + 2} \] \[ \ldots \ldots 10.5 \]
From the above table \( \Delta P = \frac{\Delta P}{\Delta W} \times 5 \ldots \ldots 10.6 \)

\( I \), for the beam used in the experiment, is 342 (mm)\(^4\), therefore, using equations 10.5 and 10.6 and substituting the values of \( I, k \) and \( L \):

\[
E = 0.625 \times 3 \times 1000^3 - 2 \times \frac{3 x 1000^3}{6 \times 342} \\
\frac{3.125 - 2}{\Delta P} \times 1.46199 \times 10^6 \text{ N/mm}^2
\]

The result from step 11 of the procedure (i.e. carrying out experiment 8) will give another value for the Modulus of Elasticity which should be in close agreement with the value obtained above.
BENDING OF BEAMS ABOUT A PRINCIPAL AXIS

<table>
<thead>
<tr>
<th>Deflection at 'x'</th>
<th>End Slope</th>
<th>Reaction Moment</th>
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<td>$WL^3$</td>
<td>$WL^2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Wa^2b^2$</td>
<td>$Wab(L+b)$</td>
<td>$6EIL$</td>
</tr>
<tr>
<td>$3EIL$</td>
<td></td>
<td></td>
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<tr>
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<tr>
<td>$5wl^4$</td>
<td>$wl^3$</td>
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<td>$384EI$</td>
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<tr>
<td>$wl^4$</td>
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<td>$wl^2$</td>
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<td>$384EI$</td>
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<td>$12$</td>
</tr>
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<tr>
<td>$wl^3$</td>
<td>$wl^2$</td>
<td>$wl$</td>
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<td>$6EI$</td>
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Packing Contents List

Product: SM104 BEAM APPARATUS

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<th>DESCRIPTION</th>
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<td>Dial Gauge Assembly</td>
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<td>2</td>
<td>4</td>
<td></td>
<td>Weight hanger Assembly</td>
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</tr>
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<td>3</td>
<td>1</td>
<td>25453</td>
<td>Cantilever Support</td>
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<tr>
<td>4</td>
<td>3</td>
<td></td>
<td>Load Cell Assembly with Dial Gauges and Locked Knife Edge</td>
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<tr>
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<td>Tee Leg Assembly with 3 Fixing Bolts/Nuts</td>
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<td>25486</td>
<td>Front Horizontal Member</td>
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<td>5</td>
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<td>Specimen Beams</td>
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<td>TQDS1</td>
<td>Data Sheet for Activated Clay Desiccant</td>
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<td>25551</td>
<td>2N Weight</td>
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<td>Force Spring Balance Assembly</td>
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<td>19</td>
<td>1</td>
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<td>Test Certificate</td>
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Signed: ____________________

Sheet 1 of 1
# TECHNICAL DATA SHEET

<table>
<thead>
<tr>
<th>Product Name:</th>
<th>ACTIVATED CLAY DESICCANT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nature of Material:</td>
<td>GRANULAR (PACKED IN PAPER OR NON-WOVEN FABRIC BAGS)</td>
</tr>
<tr>
<td>Chemical Name:</td>
<td>NATURAL ALUMINA SILICATE</td>
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</tbody>
</table>

## Chemical Composition:

<table>
<thead>
<tr>
<th>Composition</th>
<th>Content</th>
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</thead>
<tbody>
<tr>
<td>SiO₂</td>
<td>60%</td>
</tr>
<tr>
<td>Al₂O₃</td>
<td>20%</td>
</tr>
<tr>
<td>Fe₂O₃</td>
<td>5.5%</td>
</tr>
<tr>
<td>MgO</td>
<td>3.5%</td>
</tr>
<tr>
<td>CaO</td>
<td>2%</td>
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<tr>
<td>Na₂O</td>
<td>1%</td>
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<tr>
<td>Loss on</td>
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<tr>
<td>Ignition</td>
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## HEALTH HAZARD INFORMATION

<table>
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<tr>
<th>Condition</th>
<th>Effect</th>
<th>Recommended First Aid</th>
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<tbody>
<tr>
<td>Ingestion</td>
<td>Non-toxic - no harmful effects known</td>
<td></td>
</tr>
<tr>
<td>Inhalation</td>
<td>No harmful effects known</td>
<td>Wash with water</td>
</tr>
<tr>
<td>Contact with skin</td>
<td>No harmful effects known</td>
<td></td>
</tr>
<tr>
<td>Contact with eyes</td>
<td>No harmful effects known but may be irritating as a foreign body</td>
<td>Rinse with water</td>
</tr>
</tbody>
</table>

## FIRE HAZARD INFORMATION

<table>
<thead>
<tr>
<th>Flammability</th>
<th>Unpacked Activated Clay is non-flammable</th>
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</thead>
</table>

## HANDLING, STORAGE, SPILLAGE

<table>
<thead>
<tr>
<th>Recommendations for Safe Handling &amp; Storage</th>
<th>Store inside warehouse - keep dry.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recommendations for Dealing with Spillage and Waste</td>
<td>No special precautions. Collect into container and dispose of, according to local and national regulations</td>
</tr>
</tbody>
</table>
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<table>
<thead>
<tr>
<th>Name:</th>
<th>Telephone No:</th>
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<tr>
<td>Address:</td>
<td>Postcode:</td>
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</table>

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- Interactive Video
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