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2. Fluids

2.1. General Characteristics
2.2. Dispersions
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2.4. Transport Phenomena

2.4.1. Diffusion

2.4.2. Viscosity

2.4.3. Transport of Heat

2.4.4. Characteristic Numbers
2.4 Summary of Phenomenological Laws

- Transport processes
  - Arbitrary thermal motion on molecular level
  - Driven by gradients (inhomogeneities)
  - Mostly linear relationship

\[
\text{Flow} = \text{coefficient} \times \text{force}
\]

<table>
<thead>
<tr>
<th>effect</th>
<th>transported</th>
<th>gradient</th>
<th>coefficient</th>
<th>law</th>
</tr>
</thead>
<tbody>
<tr>
<td>diffusion</td>
<td>(N)</td>
<td>(\frac{dN}{dz})</td>
<td>(D \approx \frac{1}{3} \rho v_T l_{mfp}) (diffusion coeff.)</td>
<td>(j_N = -D \nabla N) (Fick)</td>
</tr>
<tr>
<td>viscosity</td>
<td>(mv_z)</td>
<td>(m \frac{dv_z}{dx})</td>
<td>(\eta \approx \frac{1}{3} \rho v_T l_{mfp}) (viscosity)</td>
<td>(j_{p,x} = -\eta \frac{dv_z}{dx}) (Newton)</td>
</tr>
<tr>
<td>conduction of heat</td>
<td>(Q)</td>
<td>(\rho C_m \frac{dT}{dz})</td>
<td>(\lambda \approx \frac{1}{3} \rho C_m v_T l_{mfp}) (therm. conduct.)</td>
<td>(j_Q = -\lambda \nabla T) (Fourier)</td>
</tr>
<tr>
<td>electric conductivity</td>
<td>(q)</td>
<td>(-\frac{d\phi}{dz} = E_z)</td>
<td>(\sigma_E \approx \frac{q q^2 l_{mfp}}{m v_T}) (electr. conduct.)</td>
<td>(j_q = -\sigma E \nabla \phi) (Ohm)</td>
</tr>
</tbody>
</table>

**Table 2.1.** Summary of phenomenological laws of transport and coefficients calculated for ideal gases. For the viscosity, the \(z\)-direction delineates the direction of flow and \(x\) the transversal axis.
2.4.1. Diffusion

• Diffusion
  - Counteracts nonuniform particle densities
  - Thermal „Brownian“ motion
  - Process underlying all other transport phenomena in fluids

• Fick‘s first law

\[
j_N = -D \nabla q_N
\]

- Current density \( j_N \) antiparallel to gradient
- Systems seeks homogeneity
- Diffusion coefficient \( D \) [\( m^2 \) s\(^{-1}\)]
2.4.1. Diffusion

- Fick‘s second law
  - Time domain
  - Fixed location

\[
\frac{\partial \rho_N}{\partial t} = D \Delta \rho_N
\]

- Combining Fick‘s first law and equation of continuity

\[
\frac{\partial \rho_N}{\partial t} = -\nabla \cdot j_N
\]

- Laplace equation

\[
\Delta \rho_N = 0
\]

  - Stationary conditions
2.4.1 Molecular Picture

• Derivation of diffusion constant $D$

• Flow through surface $z_1$
  - Between two planes with diverging particle densities
  - Distance = mean free path $l_{\text{mfp}}$
  - Particles reach plane without collisions (statistically)

![Net flow through surface located at $z = z_1$](image)

\[ j_{N,z} = j_{N,+z} - j_{N,-z} = \frac{1}{2} [\rho_N(z - l_{\text{mfp}}) - \rho_N(z + l_{\text{mfp}})] \bar{v}_z \]

\[ j_{N,z} \approx - \frac{d\rho_N}{dz} \bar{v}_z l_{\text{mfp},z} \]

- Linear term of Taylor expansion

\[ j_N = -D \nabla \rho_N \]
2.4.1 Diffusion coefficient

- Calculation of diffusion coefficient
  - Assumption: uniformly distributed directions of all vectors $\nu$

$$D = \langle \bar{v}_z l_{mfp,z} \rangle = v_T l_{mfp} \frac{1}{2\pi} \int_0^{\pi/2} \cos^2 \Theta 2\pi \sin \Theta d\Theta$$

\[D = \frac{1}{3} v_T l_{mfp}\]

<table>
<thead>
<tr>
<th>solute</th>
<th>solvent</th>
<th>$D / 10^{-9} \text{m}^2\text{s}^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I$_2$</td>
<td>C$<em>6$H$</em>{12}$</td>
<td>4.05</td>
</tr>
<tr>
<td>I$_2$</td>
<td>CCl$_4$</td>
<td>3.42</td>
</tr>
<tr>
<td>I$_2$</td>
<td>C$_6$H$_6$</td>
<td>2.13</td>
</tr>
<tr>
<td>N$_2$</td>
<td>CCl$_4$</td>
<td>3.42</td>
</tr>
<tr>
<td>O$_2$</td>
<td>CCl$_4$</td>
<td>3.82</td>
</tr>
<tr>
<td>Ar$_2$</td>
<td>CCl$_4$</td>
<td>3.63</td>
</tr>
<tr>
<td>H$_2$O</td>
<td>H$_2$O (self diffusion)</td>
<td>2.62</td>
</tr>
<tr>
<td>dextrose</td>
<td>H$_2$O</td>
<td>0.67</td>
</tr>
<tr>
<td>H$^+$</td>
<td>H$_2$O</td>
<td>9.31</td>
</tr>
<tr>
<td>Li$^+$</td>
<td>H$_2$O</td>
<td>1.03</td>
</tr>
<tr>
<td>Na$^+$</td>
<td>H$_2$O</td>
<td>1.96</td>
</tr>
<tr>
<td>Cl$^-$</td>
<td>H$_2$O</td>
<td>2.03</td>
</tr>
<tr>
<td>Br$^-$</td>
<td>H$_2$O</td>
<td>2.08</td>
</tr>
<tr>
<td>I$^-$</td>
<td>H$_2$O</td>
<td>2.05</td>
</tr>
</tbody>
</table>

*Table 2.1. Diffusion coefficients $D$ of molecules and ions in various solvents*
2.4.1 Examples for Diffusional Processes

- Example
  - Bilateral diffusion from 2-D layer
  - Within pure solvent
  - Initial conditions

$$\rho_N(t = 0, z = 0) = \rho_{N,0}$$

$$\rho_N(t = 0, z \neq 0) = 0$$

- Solution by "educated guess"

$$\rho_N(z, t) = \frac{\rho_{N,0}}{\sqrt{\pi D t}} e^{-\frac{z^2}{4Dt}}$$

Graph showing particle concentration over diffusional distance and time.
2.4.1 Examples for Diffusive Processes

![Diagram showing diffusion process with concentration and length axes, and a value of $D = 3 \times 10^{-9}$ m$^2$s$^{-1}$]
2.4.1 Examples for Diffusive Processes
2.4.1 Examples for Diffusive Processes

![Diffusion Diagram]

- **Diffusion**
- **Concentration c** vs **Length x [µm]**
- **Time** from 0s to 5s
- **D = 3 \times 10^{-9} m^2 s^{-1}**
2.4.1 Examples for Diffusive Processes
2.4.1 Examples for Diffusive Processes

![Diagram of diffusion process with concentration and time parameters.](image)

\[ D = 3 \times 10^{-9} \text{ m}^2\text{s}^{-1} \]
2.4.1 Examples for Diffusive Processes

- Example
  - Diffusion through permeable diaphragm
  - Initial conditions

\[ \rho_N(t = 0, z < 0) = \rho_{N,0} \]
\[ \rho_N(t = 0, z > 0) = 0 \]

**Example**

- Diffusion through permeable diaphragm
- Initial conditions

**Fig. 2.9.** Diffusion of molecules through a permeable wall at \( z = 0 \) which are initially located at \( z < 0 \). \( D = 3.0 \times 10^{-9} \text{ m}^2 \text{ s}^{-1} \) has been assumed. With increasing time, a uniform distribution builds out.
2.4.1. Laminar Mixing
2.4.1 Time and Length Scales

- **Distance**

\[ l_D = \sqrt{2Dt} \]

- **Time**

\[ t_D = \frac{l^2}{2D} \]

(“rules of thumb”)

<table>
<thead>
<tr>
<th>( l )</th>
<th>( V )</th>
<th>( t_D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 mm</td>
<td>1 μl</td>
<td>3.5 min</td>
</tr>
<tr>
<td>100 μm</td>
<td>1 nl</td>
<td>3 s</td>
</tr>
<tr>
<td>10 μm</td>
<td>1 pl</td>
<td>33 ms</td>
</tr>
</tbody>
</table>

**Table 2.8.** Typical diffusion time \( t \) for lengths \( l \) and corresponding volumes \( V \) for \( D = 3 \times 10^{-9} \text{ m}^2 \text{ s}^{-1} \)
2.4.1 Brownian Motion

- Discovered by Scottish Botanist Robert Brown in 1827
  - Random thermal motion of pollen under microscope
- Closely related to thermal velocity
  \[ v_T = \sqrt{\bar{v}^2} = \sqrt{\frac{3k_B T}{m}} \]
- Mesoscopic particle
  - \( r_0 = 1 \ \mu m \)
  - \( m = 10^{-15} \ \text{kg} \)
  - \( T = 293 \ \text{K} \)
  - \( v_T \approx 3 \ \text{mm s}^{-1} \)

Fig. 2.10. Brownian motion resulting from random molecular pressure oscillations leads to wobbling motion of mesoscopic particles that can be observed under a microscope

\[ z^2 = 3Dt = \frac{k_B T}{2\pi \eta r_0}t \]

Width of distribution of particle locations
2.4.1 Brownian Motion

- Fluorescent particles
- 4 seconds of data
- 2 µm in diameter
- Left picture
  - Particles moving in pure water
- Right picture shows
  - Particles moving in concentrated solution of DNA
  - I.e., in viscoelastic solution in other words

From http://www.deas.harvard.edu/projects/weitzlab/research/brownian.html
2.4.1 Brownian Motion

- Erratic motion of single milk droplets in water
- Droplets size of about 1 micrometer
- Continuously kicked by fast-moving water molecules
  - Size 5000 times smaller
- Magnification factor of about 10,000

http://www.microscopy-uk.org.uk/
2.4.1 Boltzmann Distribution

- System at $T$ in thermal equilibrium
- Each particle with certain kinetic energy
- Potential gradient
  - Particles accumulate in potential minimum
- Counteracting diffusive current
  \[ j_N = -D \nabla \rho_N \]
- Boltzmann distribution
  - Ratio of particle densities $\rho_{Ni}$ at two locations separated by potential difference $\Delta E$
  \[ \frac{\rho_{N,1}}{\rho_{N,0}} = e^{-\frac{\Delta E}{k_B T}} \]

Boltzmann function at $T = 273$ K calibrated to $\rho_{N0} = 1$.
2.4.1 Maxwell Distribution

- Maxwell distribution
  - Fraction of molecules in velocity interval \([\nu, \nu + d\nu]\)
  - \(f(\nu)d\nu\) is probability of finding a vector \(\nu\) in \([\nu, \nu + d\nu]\)
  - Normalization of integrand function \(f(\nu)\) to unity
    \[
    \int_0^\infty f(\nu)d\nu = 1
    \]
  - Leads to Maxwell distribution

\[
\begin{align*}
    f(\nu)d\nu &= \sqrt{\frac{2}{\pi}} \left( \frac{m}{k_B T} \right)^{3/2} \nu^2 e^{-mv^2/2k_B T}d\nu
\end{align*}
\]
2.4.1 Maxwell Distribution

- Transformation into energy space
  - Replacing $v$ with kinetic energy $E = 1/2 \, mv^2$

$$f(E) \, dE = \frac{2}{\sqrt{\pi}} \, (k_B T)^{-3/2} \, \sqrt{E} \, e^{-E/k_B T} \, dE$$

  - Helps determining which fraction of particles is capable of "jumping" over a certain potential energy barrier $E_0$
  - Analytical expression for this fraction

$$\text{tail fraction} = \frac{\int_{E_0}^{\infty} f(E) \, dE}{\int_0^{\infty} f(E) \, dE} = \frac{2}{\sqrt{\pi}} \, \sqrt{\frac{E_0}{k_B T}} \, e^{-E_0/k_B T}$$

Fig. 2.12. Maxwellian function $f(v)$ for $O_2$ gas at different temperatures
2.4.1 Reaction Kinetics

- Chemical Reactions
  - Binary collisions
  - Collision rate scales linearly with $v_T$ and thus with $T^{1/2}$
  - Concentration of reaction partners $c$ also influences speed of reaction

- For single step reaction

$$xX + yY + \ldots \rightarrow \text{products}$$

- Reaction rate

$$\frac{dn}{dt} = k'_c c^x(X)c^y(Y)\ldots$$

  - Governed by stoichiometry ($x$, $y$, $\ldots$)
    - Concentrations $c(X)$, $c(Y)$, $\ldots$
    - Reaction velocity constant $k'_c$
2.4.1 Reaction Kinetics

- Chemical reactions
  - Activation energy (Gibbs energy)
  - Simple model reaction

\[
A_2 + B_2 \rightleftharpoons (A_2B_2)^* \rightleftharpoons 2AB
\]

- For simplicity: \(\Delta G = \Delta U = E_{\text{act}}\)
- Typically \(E_{\text{act}} = 60 \text{ to } 250 \text{ kJ mol}^{-1}\)
- \(\Delta G = G_{\text{act}} - G'_{\text{act}}\)
  - \(\Delta G < 0\) exothermic
  - \(\Delta G > 0\) endothermic

- Velocity constant of reaction
  - Strongly dependent on \(T\)
  - Reaction-specific
  - Arrhenius equation

\[
k'_c(T') = k_0 e^{-E_{\text{act}}/R_g T}
\]
2.4. Transport Phenomena

2.4.1. Diffusion

2.4.2. Viscosity

2.4.3. Transport of Heat

2.4.4. Characteristic Numbers
2.4.2. Viscosity

- Viscosity
  - Transfer of momentum from one plane sliding parallel to another
  - Mediated by fluid sandwiched between them
  - „Internal friction“ of fluid

- Newton’s law of viscosity

\[ \dot{p}_x = -\eta \frac{d v_z}{dx} \]

- Relates flow of axial momentum \( p_z = m v_z \) along lateral \( x \)-direction from one plane sliding parallel to another one by viscosity \( \eta \)
- Unit of \( \eta \): Pa s = kg m\(^{-1}\) s\(^{-1}\) or old Poise with 1 Ps = 0.1 Pa s
2.4.2 Viscosity of Gases

- Assumptions for picture:
  - Flow in z-direction
  - Uniform particle density

- Net flux in x-direction of longitudinal momentum $p_z$

$$j_{p,x} = \frac{1}{2} \rho N m [v_z(x - l_{mfp}) - v_z(x + l_{mfp})] \bar{v}_x$$

$$j_{p,x} \simeq -\rho \frac{dv_z}{dx} l_{mfp} \frac{1}{3} v_T$$

- Viscosity of gases

$$\eta = D \rho = \frac{1}{3} \rho N m v_T l_{mfp}$$

- Kinematic viscosity
  - "Momentum diffusivity"

$$v = \frac{\eta}{\rho} \text{id}. \text{gas} \ D$$

Fig. 2.14. Molecular picture of the viscosity of gases. The left wall is at rest, the right wall located at $z = d$ moves at speed $v_0$ in x-direction and a linear flow profile is assumed in first approximation.
2.4.2 Viscosity of Gases

<table>
<thead>
<tr>
<th>gas</th>
<th>$\eta / 10^{-5}$ Pa·s</th>
</tr>
</thead>
<tbody>
<tr>
<td>He</td>
<td>1.96</td>
</tr>
<tr>
<td>Ne</td>
<td>2.99</td>
</tr>
<tr>
<td>Ar</td>
<td>2.08</td>
</tr>
<tr>
<td>H$_2$</td>
<td>0.85</td>
</tr>
<tr>
<td>O$_2$</td>
<td>1.91</td>
</tr>
<tr>
<td>N$_2$</td>
<td>1.67</td>
</tr>
<tr>
<td>CO$_2$</td>
<td>1.38</td>
</tr>
<tr>
<td>Air</td>
<td>1.71</td>
</tr>
</tbody>
</table>

*Order: $10^{-5}$ Pa·s*

Table 2.9. Viscosity of gases at $T = 293$ K
2.4.2 Viscosity of Liquids

- Liquid
  - Very dense gas
  - $l_{mfp} \approx$ average distance between molecules
  - Macroscopic behavior governed by intermolecular potentials

- Picture
  - Moving A to A'
  - Activation energy $E_0$
  - External shear stress $\sigma_{xz}$
    - Force $f$ on each particle on wall
    - $f$ counteracts motion in negative z-direction
    - Momentum loss transferred to B
    - "Activation energy" $\Delta E = f \ a / 2$

Fig. 2.15. Molecular model for the viscosity of liquids
2.4.2 Viscosity of Liquids

- Transfer of momentum $\Delta p_Z = m \Delta v_Z$
  - Lateral velocity gradient
    \[
    \frac{\Delta v_z}{\Delta x} = a \nu_{\text{net}} / a = \nu_{\text{net}}
    \]
  - $\nu_{\text{net}}$: difference between number of jumps $\nu_{+Z}$ and $\nu_{-Z}$ in positive and negative $z$-direction, respectively
  - Evaluation of gradient (using Boltzmann Ansatz)
    \[
    \frac{\Delta v_z}{\Delta x} = \nu_{+z}^\text{net} = \nu_{+z} - \nu_{-z}
    \]
    \[
    = \nu_0 \exp \left(- \frac{E_0 - \Delta E}{k_B T} \right) - \nu_0 \exp \left(- \frac{E_0 + \Delta E}{k_B T} \right)
    \]
    \[
    = \nu_0 \exp \left(- \frac{E_0}{k_B T} \right) \exp \left( \frac{\Delta E}{k_B T} \right) \left[ \exp \left( \frac{\Delta E}{k_B T} \right) - \exp \left(- \frac{\Delta E}{k_B T} \right) \right]
    \]
    \[
    \approx \nu_0 \exp \left(- \frac{E_0}{k_B T} \right) 2 \frac{\Delta E}{k_B T}
    \]
    \[
    \frac{\Delta v_z}{\Delta x} = 2\nu_0 \frac{fa}{2k_B T} \exp \left(- \frac{E_0}{k_B T} \right)
    \]
    $\nu_0$: fundamental oscillation frequency of molecule between neighbors

Fig. 2.15. Molecular model for the viscosity of liquids
2.4.2 Viscosity of Liquids

- First order Taylor expansion for $\Delta E / k_B T$

$$\exp\left(\frac{\Delta E}{k_B T}\right) \simeq 1 + \frac{\Delta E}{k_B T}$$

- Viscosity of liquids

$$\eta = \frac{\sigma_{xz}}{\Delta v_z / \Delta x} = \frac{f/a^2}{2\nu_0(fa/2k_B T) \exp(-E_0/k_B T)}$$

$$\eta = \frac{k_B T}{a^3 \nu_0} \exp\left(\frac{E_0}{k_B T}\right)$$

$T \downarrow$
### 2.4.2 Viscosity of Liquids

#### Table 2.10. Viscosity of selected liquids at different temperatures

<table>
<thead>
<tr>
<th>liquid</th>
<th>T/K</th>
<th>η / 10^{-3} Pa s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hg</td>
<td>298</td>
<td>1.53</td>
</tr>
<tr>
<td>CCl₄</td>
<td>298</td>
<td>0.909</td>
</tr>
<tr>
<td>C₆H₆</td>
<td>298</td>
<td>0.601</td>
</tr>
<tr>
<td>C₆H₆</td>
<td>323</td>
<td>0.898</td>
</tr>
<tr>
<td>C₆H₆</td>
<td>348</td>
<td>0.664</td>
</tr>
<tr>
<td>C₆H₆</td>
<td>373</td>
<td>0.521</td>
</tr>
<tr>
<td>CO₂</td>
<td>273</td>
<td>0.098</td>
</tr>
<tr>
<td>H₂O</td>
<td>298</td>
<td>0.890</td>
</tr>
<tr>
<td>glycerin</td>
<td>293</td>
<td>1500</td>
</tr>
<tr>
<td>ethanol</td>
<td>293</td>
<td>1.21</td>
</tr>
<tr>
<td>benzol</td>
<td>293</td>
<td>0.65</td>
</tr>
<tr>
<td>ethylic ether</td>
<td>293</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Order: 10^{-3} Pa s
(gases: 10^{-5} Pa s)
2.4.2 Newtonian Fluids

- Newtonian fluids
  - Classical idealized fluids
  - Linear stress-strain relationship
  - I.e. linear relationship between tension tensor $\sigma$ and tensor of rate of deformation (strain rate) $\nu$
    
    $\sigma \propto \nu$
  
  - Viscosity $\eta$: constant of proportionality
  - Approximated by gases and "simple" liquids like water

- Non-Newtonian fluids
  - Dispersions containing large structured macromolecules
  - E.g. polymers and proteins
2.4. Transport Phenomena

2.4.1. Diffusion

2.4.2. Viscosity

2.4.3. Transport of Heat

2.4.4. Characteristic Numbers
2.4.3. Transport of Heat

- Equilibrium thermodynamics
  - System seeks uniform temperature distribution
- Inhomogeneous $T$-distribution
  - Transport of heat
- Two basic modes
  - Diffusion
    - Statistical phenomenon (related to entropy)
    - Summarized in macroscopic thermal conductivity $\lambda$
  - Convection
    - Far more complicated
    - Macroscopic ramifications
    - Minor importance in microworld
- Another frequently encountered issue
  - Transfer of thermal energy at interface between two media/phases
  - No inter-phase diffusion / exchange of particles
  - Transmission and transition of heat
2.4.3 Thermal Conductivity

- Fourier's law
  - Basic equation quantifying diffusive transport of heat
    \[ j_Q = -\lambda \frac{dT}{dz} \]
  - Net flow of energy \( j_Q \)
  - Thermal conductivity \( \lambda \)

- Power
  \[ P = j_Q A = A\lambda \frac{dT}{dz} \]
  - Cross-section of flow \( A \)

- Relaxation time for temperature gradient
  - Distance \( d \)
2.4.3 Thermal Conductivity

- Molecular picture

- Net energy flow in positive z-direction

- Using linear terms of Taylor-expansion

\[
\dot{Q}_{+z} = \frac{1}{6} \rho_N v_T \left[ \bar{E}_N(z - l_{mf}) - \bar{E}_N(z + l_{mf}) \right]
\]

\[
= \frac{1}{6} \rho_N v_T \left\{ \left[ \bar{E}_N(z) - l_{mf} \frac{\partial \bar{E}_N}{\partial z} \right] - \left[ \bar{E}_N(z) + l_{mf} \frac{\partial \bar{E}_N}{\partial z} \right] \right\}
\]

\[
= -\frac{1}{3} \rho_N v_T l_{mf} \frac{\partial \bar{E}_N}{\partial z}
\]

\[
\dot{Q}_{+z} = -\left( \frac{1}{3} \rho_N v_T l_{mf} \frac{\partial \bar{E}_N}{\partial T} \right) \frac{\partial T}{\partial z} = -\lambda \frac{\partial T}{\partial z}
\]
2.4.3 Thermal Conductivity

- Thermal conductivity

\[ \lambda = D \rho C_m = \frac{1}{3} \rho C_m v_T l_{\text{mfp}} \]

- Approximating on \( l_{\text{mfp}} \)

\[ \lambda = \frac{1}{3 \sqrt{2} \sigma_{\text{coll}}} v_T \frac{\partial \tilde{E}_N}{\partial T} \]

- Independent of particle density \( \rho_N \) for ideal gas

- Heat diffusion coefficient

\[ \lambda_D = \frac{\lambda}{\rho C_m} \text{id.gas} \]

\[ v = \frac{\eta}{\rho} \text{id.gas} \Rightarrow D \text{ kinematic viscosity} \]
2.4.3 Thermal Conductivity

- Typical values

<table>
<thead>
<tr>
<th>gas</th>
<th>$\lambda / \text{W m}^{-1}\text{K}^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>He</td>
<td>$1.44 \times 10^{-1}$</td>
</tr>
<tr>
<td>Ne</td>
<td>$4.60 \times 10^{-3}$</td>
</tr>
<tr>
<td>Ar</td>
<td>$1.79 \times 10^{-3}$</td>
</tr>
<tr>
<td>H$_2$</td>
<td>$1.75 \times 10^{-1}$</td>
</tr>
<tr>
<td>O$_2$</td>
<td>$2.43 \times 10^{-2}$</td>
</tr>
<tr>
<td>N$_2$</td>
<td>$2.39 \times 10^{-2}$</td>
</tr>
<tr>
<td>CO$_2$</td>
<td>$1.42 \times 10^{-2}$</td>
</tr>
<tr>
<td>air</td>
<td>$2.41 \times 10^{-2}$</td>
</tr>
<tr>
<td>H$_2$O</td>
<td>$1.58 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

*Table 2.11. Thermal conductivity of gases at $T = 273$ K*

<table>
<thead>
<tr>
<th>material</th>
<th>temperature $\theta / ^\circ\text{C}$</th>
<th>thermal conductivity $\lambda / \text{W m}^{-1}\text{K}^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>aluminum</td>
<td>0 – 200</td>
<td>230</td>
</tr>
<tr>
<td>silica glass</td>
<td>0 – 100</td>
<td>1.4</td>
</tr>
<tr>
<td>helium</td>
<td>0</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.17</td>
</tr>
<tr>
<td>air</td>
<td>0</td>
<td>0.024</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.031</td>
</tr>
<tr>
<td>water</td>
<td>0</td>
<td>0.54</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.67</td>
</tr>
<tr>
<td>ethanol</td>
<td>0</td>
<td>0.18</td>
</tr>
</tbody>
</table>

*Table 2.12. Thermal conductivity for solids, liquids and gases*
2.4.3 Convection

• Macroscopic transport of particles
  ➢ Characteristic for macro-devices
  ➢ Important for heat transport in liquids and gases

• Forced convection
  ➢ Transport of heat by macroscopic particle flow
  ➢ E.g., by an *external mechanical* source

• Free convection
  ➢ For instance driven by *buoyancy*
2.4.3 Convection

- Convective currents
  - Stationary or non-stationary flow patterns

- Simulation of convection
  - Often fail to coincide with experimental observations
  - E.g., atmospheric weather and climate

- Free Convection of minor importance in microdevices
  - Laminar conditions
2.4. Transport Phenomena

2.4.1. Diffusion
2.4.2. Viscosity
2.4.3. Transport of Heat

2.4.4. Characteristic Numbers
2.4.4. Characteristic Numbers

• Fourier mass number
  ➢ Characterizes diffusion

\[
Fo_m = \frac{Dt}{l^2}
\]

➢ Time \( t \), e.g. residence in chemical reaction chamber
➢ Typical diffusive time scale \( t_D = \frac{l^2}{D} \)

• Schmidt number
  ➢ Relates viscosity and diffusion

\[
Sc = \frac{\eta}{\varrho D} = \frac{\nu}{D} \quad \text{id. gas} \quad 1
\]

➢ Roughly 0.8 for gases
2.4.4. Characteristic Numbers

- Fourier number
  - Diffusion of heat
  - Stored thermal energy

\[ F_0 = \frac{\lambda t}{\rho C_m l^2} = \frac{\lambda_D t}{l^2} \]

- Prandtl number
  - Ratio between momentum diffusivity (via dynamic viscosity \( \eta \)) and heat diffusivity (via thermal conductivity \( \lambda \))

\[ Pr = \frac{C_m \eta}{\lambda} = \frac{\nu}{\lambda_D} \overset{\text{id.gas}}{=} 1 \]

- Specific heat capacity \( C_m \)
- Typically 3 to 300 for liquids and 0.7 to 1.0 for gases