Performing Cosmic Music: Notes on Plato's Timaeus

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'Music is existence.' (Sun Ra)

In this paper, I want to outline how 'sacred music' and 'performance' might be understood in the broadest possible terms—as nothing less than a description of reality itself. This may seem a grandiose—not to say ridiculous—claim; nonetheless, by concentrating on the *Timaeus*, I want to show that, for Plato, the cosmos is musically 'constituted' by a divine creator and that, as such, its fundamental operation is to be understood as a mathematico-harmonic *performance*. Furthermore, and as I shall also try to show, Plato goes on to posit this cosmo-musical order as a model for the 'attunement' of the individual soul: cosmic *harmonia*, he suggests, has a thoroughly moral significance, inasmuch as it provides the exemplar for how we should comport ourselves, individually and socially.

My paper has five main components. First, it provides a general survey of the *Timaeus* and the issues that it addresses. Secondly, it focuses on the Mind, or 'World-Soul', that Plato depicts as forming the intellectual core of the cosmos; here, I give particular focus to its mathematical *and musical* infrastructure. With this established, I turn, thirdly, to the way in which Plato locates the human being as the micro-cosmic counterpart of this wider order, and the way in which the 'harmony of the spheres' is thus revealed as a moral propaedeutic. Fourthly, I go on to outline something of the legacy of Plato's conception of a *harmonia mundi*, indicating the vast range of thought (Jewish and Islamic, as well as pagan and Christian) that it influences. Finally, I try to emphasise that—regardless of its formative role for so much philosophical,

theological and scientific speculation—the *Timaeus* retains a certain caution regarding its own claims, and that this is crucial in whatever we might take from Plato's text.

Plato's Timaeus: A Very Short Overview

The *Timaeus* is one of Plato's strangest works; but it is also one of his most significant, at least in terms of historical effect and influence. The most basic description we could give is that it presents Plato's cosmology; in part, it is Plato's contribution to an established tradition, or genre, of treatises 'on nature' (*peri phuseôs*). However, given the scope of Plato's dialectical ambition, the *Timaeus* amounts to far more than a 'neutral' description of astronomical phenomena: as we shall see, it also presents, or at least suggests, what we could term a moral anthropology. And, as a central part of this wide-ranging exploration, it raises profound questions about the ontological status of music—questions which form the main focus of this paper.

In summary, we could say that the *Timaeus* gives us a creation account presented in Pythagorean terms: cosmic order is the result of mathematical proportion and harmony. Its basic question is about the *genesis* of the cosmos: is it eternal, or was it created? Its basic answer is that the cosmos must have been 'made', and thus that it must presuppose an intelligent maker whom we can (sort of) understand as a divine craftsman, a *Demiurgos*.¹ This Demiurge—who is also referred to as a 'father' (*pater*), a 'manufacturer' (*poietes*), and an 'arranger' (*suntheis*)— shapes the cosmos according to something like an eternal blueprint: he works in reference to, and is guided by, conceptual paradigms and foundational mathematical principles. But he does not produce, or create, *ex nihilo*: the basic, formless, 'stuff' of the cosmos precedes the Demiurge's

¹ The notion of a Demiurge who orders the heavens had already been adumbrated in the *Republic* (530a).

labours—even if the former only assumes a recognisable shape thanks to the latter.² What is more, the Demiurge does not just shape the *physical* stuff of the universe: he also forms a 'soul' for this cosmic body, a governing intelligence that provides rational regulation and order (even if, in Plato's account, the question of chronological seniority remains unsettled: see 34c). Nothing without intellect could ever be finer or more beautiful than that which has intellect, Plato declares (30b); so, by conjoining matter and intelligence—or, rather, by *informing* the physical domain with a World-Soul—the Demiurge ensured that he "fashioned a work that would be most beautiful [*kalliston*] and best [*ariston*]" (30b).

The *Timaeus* is thus able to bring to a (kind of) conclusion a long-standing concern of Plato's. The *Phaedo*, from a much earlier point in his oeuvre, tells us that the young Socrates was converted from a physicalist understanding by Anaxagoras' claim that *nous*, or mind, is the foundational principle of all that *is* and the 'initiator' of the cosmos (97b–99d); the same dialogue also makes plain, however, that Anaxagoras failed fully to unfold the implications of his own insight, that he reverted to mechanistic explanations, and that he confused a cause with its concomitant conditions. The *Phaedrus*, meanwhile, expanding the 'psychological' account of the *Republic*, suggested that the world itself is an ensouled, living, being (*Phaedrus* 245c–249d). Now, in the *Timaeus*, Plato takes the topic even further, and provides a far more precise—although still mythological—account of the 'nature' of the World-Soul. Something like a 'Mind' is at work in the universe, regulating its course and movement; our world should thus be regarded as emanating from and expressing a supreme rationality. Furthermore, and even more importantly for this article, Plato also wants to claim that this Mind, or 'World-Soul', needs to be conceived (or at least imagined) as being *musical*—not just in analogous or illustrative terms, but

² More specifically: the basic 'elements' of fire, water, air and earth are said to have an amorphous existence, prior to the Demiurge's manufacturing, and within a vast 'receptacle' [*hupodochē*], which is also spoken of as a 'mother' and a 'nurse'.

at a foundational, ontological, level. This aspect of the World-Soul thus deserves more focussed attention.

The Musical World-Soul

Unfortunately, Plato's account of the Demiurge's composition of the World-Soul is one of the most difficult and perplexing in a work that is hardly short on difficult and perplexing accounts. Nonetheless, we could summarise, fairly crudely, as follows: The Demiurge combines three basic principles, Sameness, Difference and Existence, into a composite (and wholly conceptual) 'stuff' (35a).³ The resultant amalgam is then formed into two strips which are laid over each other to form a *chi*-like shape (χ) and then linked at their ends and rendered as intersecting circles (36c). This spherical structure, in turn, is taken to provide a celestial *logos*: it explains the regularity of the (seven known) planets' motions and cycles, the course of the sun, the different faces revealed by the moon, the nature of solstices, equinoxes, and so on. Mercifully, the fuller details of Plato's astronomy do not need to concern us, here. Instead, our main focus is on what we might term the 'mathematical infrastructure' of the astronomical description—that is, on the nature of the basic (although invisible) 'stuff' of the World-Soul.

What Plato suggests (at 35b–36b) is that each of the two basic strips of 'soul-stuff' that go on to provide the order of the heavens is crafted according to precise geometric ratios: his astronomy is rooted in Pythagoreanism.⁴ The division of each strip, Plato says, begins with 1, the number that is both odd and even, limited and unlimited; and each strip consists of a *tetractys*, a sequence of four principal numbers that, as 'four-fold', coincides with the four principal elements and the four seasons. The first, even, sequence proceeds by doubling its 'first-named':

³ For illuminating commentary on this difficult notion, see Gadamer 1980.

⁴ For a general introduction to Pythagorean thought, see McKirahan 2010, ch. 9 (79–111). For particular focus on Plato and Pythagorean thought, see McClain 1978.

1 (x2) = 2 (x2) = 4 (x2) = 8. The second, odd, sequence, proceeds by trebling its 'first-named': 1 (x3) = 3 (x3) = 9 (x3) = 27. In mathematical terms, this overall sequence—1, 2, 3, 4, 8, 9, 27—is significant in that the last number is also the sum total of the six that precede it; cosmologically, the sequence of seven replicates and reflects the number of (known) planets, and is also taken proportionately to describe the distance between them. More than all of this, however, the sequence also delineates, or codifies, a *musical scale* (ideally suited, presumably, for the seven-stringed lyre): translated into pitches, the numbers here represent a span of four octaves and a major sixth. The 'World-Soul' is *essentially* musical; it is characterised, Plato insists, by harmony (*harmonia*) as much as it is by reasoning (36e–37a).

And this is not the full extent of the Demiurge's musico-mathematical craftsmanship. Having provided this basis, he goes on to fill in certain blanks, by locating both the *harmonic* mean and the *arithmetic* mean in each of the two series. The first, the harmonic, can be represented schematically as 2ab/a+b; the second, the arithmetic, as a+b/2. Put otherwise, and borrowing Cornford's illustration (Cornford 1997, 71n.), we could say that, for example, the harmonic mean of 6 and 12 is 8 (because it exceeds 6 by the same *proportion*—a third—by which 12, in turn, exceeds it), while the arithmetic mean of the same two figures would be 9 (because it exceeds 6 and is short of 12 by one and the same *number*, 3). The result, obviously enough, is a far fuller set of subdivisions. As Plato himself describes this:

[The Demiurge] filled up the double and triple intervals by cutting off further portions from the mixture [of 'soul-stuff'] and inserting them into the gaps, so that in each interval there were two means, a mean that exceeded one of its extremes by the same fraction of the extremes as it was exceeded by the other extreme, and another mean that exceeded one of its extremes by the same number as it was exceeded by the other extreme. These links created, within the first set of intervals, further intervals of 3:2, 4:3 and 9:8, and then he filled up all the 4:3 intervals with intervals with the 9:8 interval, leaving in each case a portion, and the portion that remained was an interval whose terms, expressed numerically, were 256:243 (*Timaeus* 36a–b).

What emerges, given these further subdivisions (but without repeating numbers that occur in both of the two series), is the following sequence: 1, 4:3, 3:2, 2, 8:3, 3, 4, 9:2, 16:3, 6, 8, 9, 27:2, 18, 27. And when these figures, in turn, are 'translated' into their corresponding pitches, we find that Plato has added intervals of a fifth (3:2) and a fourth (4:3), as a well as a tone, to the original seven-note series. By doing so, he is able to provide a diatonic scale that has a (kind of) natural conclusion, or resolution.

In terms of ancient musicology, this is far from insignificant: it is Plato's attempt to overcome the problem of the so-called Pythagorean *komma*, or the difference between *apotme* and *limma*, whereby a sequence of perfect fifths can never 'close', and will inevitably exceed the octave. (Roughly, the problem of the *komma* describes an ineliminable discrepancy between what 'should' happen, mathematically, when a string is subdivided, and what actually occurs, sonically; accordingly, 'rational correctness' has to be tempered to achieve 'aural correctness'.⁵) In terms of Plato's philosophy, it means that the *Timaeus* is also able to 'unpack' (as it were) the suggestion provided in the 'Myth of Er', at the end of the *Republic* (617b–c), that celestial Sirens accompany the seven planets in their revolutions, and that the single notes each sings combine to form a wider, cosmic, harmony;⁶ it also provides fuller justification of the suggestions in the *Philebus* (17b–d) that pitch and rhythm are only properly understood in terms of their mathematical infrastructure. Now, it seems, the cosmos itself can be shown to be the performance of Pythagorean music.

⁵ For a remarkable study of the wider social and cultural implications of this issue, see Weber 1958: in his analysis, the fault-line between mathematical ideal and 'irrational' reality becomes paradigmatic for so much human endeavour.

⁶ For detailed discussion, see McClain 1978, ch. 4 (41–55).

The Attuned Human Soul

However, the *Timaeus* offers more than this description of stellar motions and their musicomathematical essence. As well as a cosmology and a cosmogony, Plato tries to describe *human* origins—or, rather, to make this anthropogony an intrinsic element of his wider narrative. And what this particular aspect of Plato's creation account suggests is a fundamental—indeed, an ontological—continuity between humanity and the wider whole, to the extent that the human being is regarded, quite literally, as a *micro-cosmos*.⁷

Humans, Plato tells us, are made up of the same elements that make up the universe itself. Physically, this means that, like the wider cosmos, our bodies are combinations of fire, earth, air and water (43a); we are created from the same 'stuff' as the stars. Meanwhile, in terms of the human *soul*, the basic conceptual elements that made up the World-Soul—Sameness, Difference and Existence—are again combined into a single amalgam that (literally) informs the individual. Physically and intellectually, then, we replicate and continue our macrocosmic exemplar. Moreover, and of particular importance for our survey here, Plato makes explicit that the same *musical* proportions and intervals that the Demiurge employed to shape the wider cosmos—"all three triple intervals, and the intervening means and bonds (3:2, 4:3, and 9:8)" (43d)—are also employed to shape the individual human soul: in terms of our most basic 'make-up', each of us is a manifestation, an expression, of the musical structure of the whole; each of us shares in the harmonious intellectual infrastructure of the universe itself.

This said, Plato also insists that, at the microcosmic level, the standard of quality has diminished somewhat from its original, macrocosmic, manifestation:

⁷ It seems that something like the ontological constituency of humanity—our microcosmic participation in the 'music of the spheres'—is the main concern here, rather than how our musical potential might be fully actualised. For Plato's treatment of the 'mechanics' of musical education (or *paideia*), see books 2 and 3 of the *Republic*, and also books 2 and 7 of the *Laws*. For commentary, see Anderson 1966.

[The Demiurge] turned once more to the bowl he had used previously to mix and blend the soul of the universe. He poured into it what was left of the ingredients he had used before and mixed them in the same way, with the only difference being that they were no longer as unfailingly pure as before, but were a grade or two lower in the scale of purity (*Timaeus* 41d).

Thus there is no 'simple' identity of human beings and World-Soul; if there were, so many further problems regarding individuation and particularity would surely emerge. And, given that the original 'ingredients' are no longer as pure as they were, humans are inevitably characterised by certain structural defects. We lack the self-sufficiency of the wider cosmos; *qua* physical, we are subject to decay, dissolution and death; and the darts and rushes of sensations, bodily impulses and desires constantly threaten to overcome and overwhelm us. (Some of Plato's physiological descriptions, towards the end of the dialogue, seem saturated with a dark pathos: the flesh mocks our 'higher nature', it seems. See, for example, the discussion of disease and sickness, at 82a–87e.) Furthermore, and unsurprisingly, Plato explicitly acknowledges the effects of what we could term our 'embodied fallibility' upon the musical proportions and intervals of our souls that he outlines at 43d: such is the "mighty flood" of sensations that crashes upon us (especially in our earliest years) that we are always susceptible to "disorderly movement" and "chaotic and irrational progress" (43a–b); our inherent musical order can never be completely broken, but it can be subject to "all kinds of disruption and corruption" (43e).

Nonetheless, inasmuch as we enjoy a *partial* share of the World-Soul, a kind of cosmic *telos* is still clear enough. "[I]t is from heaven [...that] our souls originally came into existence", Plato asserts (90a); and so the highest element in us "raises us up from the earth towards the heavenly region to which we are naturally akin" (*ibid*.). It is this 'divine' element within us that,

accordingly, we should seek to cultivate.⁸ By doing so, we can realise our highest, most heavenly, potential:

When a man is caught up in his appetites or his ambitions and devotes all his energies to them, the mental processes that go on inside him are bound to be restricted entirely to mortal beliefs, and he himself is bound to be completely and utterly as mortal as a man can be, since that is the part of himself that he has reinforced. But anyone who has devoted himself to learning and has genuinely applied his intelligence—which is to say, anyone who has primarily exercised his intellect—cannot fail to attain immortal, divine wisdom [...] He achieves the full measure of immortality that is possible for a human being (*Timaeus* 90b-c).

What this cultivation amounts to should be understood, ultimately, as an *attunement*. By studying the *harmonia* of the cosmos, Plato suggests, we can 'restore our nature to its original condition, by assimilating our intellect to what it is studying' (90d); in doing so, we 'remind' ourselves of a kind of pre-original harmony, and thus are far better positioned to live the best life that we can. Our proper goal is always to be 'melodic' (*emmelês*): 'attunement [*harmonia*] is an ally, provided by the Muses for the soul in its fight to restore itself to order and harmony' (47d). And, to reiterate, this is not a loose or metaphorical description of how an interior might be adjusted with an 'outer' order: as the cosmological account has insisted, the very basis of reality is musical; understanding (or, perhaps, *recollecting*) our original, divinely wrought, 'source' is thus encountering our own, essential, musicality. The well-ordered cosmos and the well-tempered human life are both to be conceived of as musical performances.⁹

The Legacy

To jaded, late-modern, eyes, all of this might seem, at best, naively optimistic, at worst, woefully Panglossian. But it is worth our while considering (in outline, at least) the remarkable influence

⁸ For discussions of the 'ethics' of the *Timaeus*, see Carone 2005, especially ch. 3 (53–78) and Broadie 2012, ch. 4 (84–114).

 $^{^{9}}$ It is by no means insignificant, not only that the overall creation myth of the *Timaeus* is 'bracketed' by two very particular descriptions—the first at 30a, the second at 92b—of basic, ontological, disorder, but more particularly that, in each case, the term used to describe said disorder is *plêmmelôs*, 'being out-of-tune', or 'being unmusical'.

that the *Timaeus* exerted, and the formative role it played in shaping so much subsequent speculation—for it became the foundation of the venerable tradition of Pythagorean musicology stretching at least from Theon of Smyrna (author of *Mathematics Useful for Understanding Plato*) to the beginnings of modernity.

Ptolemy, for example, dominated cosmic speculation, until the Renaissance, *via* his works on astronomy and astrology, the *Almagest* and the *Tetrabiblos*. But his final work, the *Harmonics*, built on a treatment of scales and intervals to produce a wide-ranging, Pythagorean, meditation on the cosmic dimensions of music: God, it seems, composed a harmonic universe; astronomy (and astrology) allow us to 'hear' this tuning.¹⁰ Aristeides, Calcidius, Macrobius, Proclus and Boethius would all produce musicological investigations that followed fundamentally similar principles (often drawn from direct commentary on the *Timaeus* itself): the cosmos demonstrated a mathematical 'tuning'; and the point of human existence was to become consonant with this musical order.

Augustine, who had argued, in his *De Ordine*, for the fundamental significance of mathematics in both music and cosmic order, developed a 'trinitarian' conception of music, in his *De Musica*, in which *musica instrumentalis*, the music we make, was related to *musica humana*, the 'harmony' of body and soul, and—ultimately—to *musica mundana*, the 'music of the spheres'; the extent of Pythagorean-Platonic influence was such that Augustine would describe a *carmen universitas*, a song of the universe, created by God according to 'perfect numbers', or *numeri judiciales*.

The legacy continued to manifest itself across hugely diverse terrain. The Irishman John Scotus Eriugena, the 'Brethren of Purity' (*Ikhwan al-Safa'*) from Basra, Al-Hasan al-Atib from Syria, the Judeo-Spanish thinkers Ibn Latif and Isaac ben Haim, the Franco-Flemish Jacobus

¹⁰ For excerpts from Ptolemy's *Harmonics*, see Godwin 1993, 21–39.

Leodiensis (Jacob of Liège): for these writers, and so many others, the essentially Pythagorean-Platonic doctrine of a 'music of the spheres', in which we find a cosmic paradigm for the bestordered earthly existence, remained a philosophical and spiritual foundation.

In Renaissance times, Marsilio Ficino, Franchino Gafori and Gioseffo Zarlino (to name only three of many important Italian thinkers on the topic) would continue to ponder the nature and significance of the *harmonia mundi*: tunings and ratios, and their celestial manifestations, retained a central position—even if a certain Orphic mysticism, largely suppressed in medieval times, was also added to the mix. And with the emergence of post-medieval science, the Platonic conception of a 'musical' cosmos, far from being abjured, in fact received its most carefully crafted exposition in the hands of one of the great founding figures of modern astronomy, Johannes Kepler.

In one respect, Kepler broke with the classical model to which even Copernicus and Galileo remained true, by denying the perfect circularity of planets' orbits, and instead demonstrating cosmic eccentricity. Nonetheless, he showed his commitment to a foundational Pythagorean insight by calculating, not only the angular velocity of the planets, but also the *pitches* that corresponded to these various velocities: for him, the latter were as important as the former. Indeed, his insight into the eccentric course of planetary circuits became, in his *Harmonice Mundi* (1619), the basis for indicating the *ranges* of notes that could be shown to correspond to the ranges of each planet's velocity; these ranges, in turn, showed in even more detail the basic harmonic perfection of the universe. According to Kepler's expanded thesis, Saturn 'sounded' as a major third, Jupiter as a minor third, Mars as a fifth, Earth as a major semitone, Venus as a minor semitone, and Mercury as an octave plus minor third. The entire symphony, in its overall temporal unfolding, might only be audible to God (given that it stretches

from the beginning of creation to the end of time); nonetheless, Kepler's findings meant that Pythagorean-Platonic convictions were apparently given further credibility, rather than rendered otiose and redundant.¹¹

Overall, it seems, we can regard the *Timaeus* as one of the foundations of our collective thought: whatever its strict scientific credibility, its historical impact has been vast. Iannis Xenakis' claim (made in the context of 'philosophy of music') that "[w]e are all Pythagoreans" (Xenakis, 2001, 202) may be hyperbolic, but it is not purely rhetorical.

Conclusion: A Likely Story

The extent of influence, as adumbrated above, helps to demonstrate the continuing suggestive force of the *Timaeus*. However, it would seem hermeneutically improper to give the impression of Plato bequeathing an authoritarian monument that, in turn, determines hundreds of years of self-assured and magisterial speculation. For whatever the nature of the subsequent discussion, the *Timaeus* itself is marked by a real reluctance regarding its own authority. It is riddled with hesitations, repetitions and deviations (so to speak), to the extent that Timaeus will even feel obliged (at 47e) to re-commence his narrative. Moreover, Plato insists that what he offers is a 'likely account' (*eikôs logos*) and even a 'likely story' (*eikôs muthos*); and this stress on the merely 'likely' character of the narrative, first stated at 29d, is repeated at least a dozen times, overall. Bearing in mind that the noun *eikôn* means 'image', 'likeness', 'figure', or 'semblance'; and bearing in mind, too, that the juxtaposition of 'image' and original is perhaps the most central of all Platonic tropes; so it seems that the creation account offered in the *Timaeus*— however plausible it might be—itself remains subject to a strict Platonic stipulation. The *eikôs logos* here is an *approximation* of 'the truth', it seems, not a definite statement: it is an account

¹¹ For extensive treatment, see Koyré 1992, especially 326–61.

that cannot maintain the kind of mathematical certainty that it 'contains'. As Gadamer has put it: "the truth which the narrative claims for itself is explicitly limited to what is 'probable' both in respect to what is presented as a story (*mythos*) and in respect to what is presented in rational argument (*logos*)" (Gadamer 1980, 159). The *Timaeus* is a remarkable provocation, for sure; but it is not supposed to be a 'final word' that answers all questions. On the contrary...

Nonetheless, even if the *Timaeus* carries a warning regarding its own claims (as well as regarding our potentiality), it is still prepared to *make* these claims—that is, to posit itself as a valid creative act depicting the greatest creative act, a valid image of a cosmic original.¹² The continuity between human and divine artistry is thereby 'confirmed' even as we are alerted to the inevitable limits of any such likely claim. Whatever the strangeness of the *Timaeus*, this suggestion of a continuity, a consonance, between humanity and a wider harmonic order remains one of the most significant aspects of its multiform legacy. For Plato, it seems, our greatest task, individually and collectively, is to imitate the divine music of the spheres—however partial and limited the results—and to appreciate that, ultimately, the Good is a question of overall *attunement*.

 $^{^{12}}$ To cite Kalkavage: "the likely story is not primarily about the cosmos but about the *making* of a cosmos" (Kalkavage 2001, 42).

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