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Characterising Jordan Homomorphisms

A Jordan homomorphism on a unital complex (associative) algebra is a linear mapping T into another algebra such that $T(x^2) = (Tx)^2$ for all x in the domain. In other words, T is a homomorphism of the canonically associated Jordan structure. Mappings of this kind have been studied for many decades by numerous authors; they became prominent in Banach algebra theory through Kaplansky's conjecture from 1970: Every surjective Jordan homomorphism between semisimple complex unital Banach algebras preserves invertibility; Kaplansky asked whether the converse is true. Despite many efforts, this question remains open. We will review some of the contributions to this problem and discuss some recent advances which clarify that, at least for C^* -algebras of real rank zero, the only possible obstruction can be the existence of a tracial state.