Lesson Study in Mathematics: Authentic Assessment of Inquiry Learning

Aisling Leavy, Mairéad Hourigan and Claire Carroll

Mary Immaculate College, Limerick, Ireland

Classrooms are complex and unpredictable learning environments. Preparing future teachers to respond to the fast changing needs of learners in mathematics classrooms is the challenge of teacher educators. In our paper, we describe the structures we have put in place to support pre-service teachers move beyond being passive recipients of educational theories to becoming critical consumers capable of designing creative and innovative pedagogical approaches. Our approach to inquiry learning in mathematics takes the form of Japanese Lesson Study carried out in partnership with primary schools. Our presentation draws on data collected from 7 years of Lesson study research carried out with 140 pre-service teachers in 28 primary classrooms in Limerick city. Insights into inquiry teaching and learning of primary level mathematics will be provided by the display of video of classroom teaching of mathematics. Video of pre-service teachers reflecting on the process of engaging in inquiry learning is pivotal also to our presentation, in part, because the challenge for us as teacher educators continues long after our pre-service teachers teach their lessons. Our challenge is how to assess their developing understandings of mathematics and mathematics pedagogy? How do we attempt to capture the multiple and interconnected facets of good teaching and planning of mathematics? We share our efforts in assessing the learning of our pre-service teachers as they engage in planning for and teaching inquiry based lessons in mathematics. We report on our attempts to capture and assess learning through the focus on our students' ability to: engage in research, link pedagogical theories to classroom practice, work collaboratively in groups, design mathematics lessons, observe learners as they engage with mathematics, diagnose difficulties, respond flexibility and thoughtfully to classroom events and reflect on their own development of mathematics content knowledge and pedagogical content knowledge.

INTRODUCTION

Teaching mathematics for *understanding* is a complex task. Competence in mathematics requires that children construct rich conceptual understandings of mathematics, develop connections between procedures, concepts and representations, and engage in dialogue and discourse around mathematics. Supporting the construction of these competencies requires that teachers themselves have rich connected understandings of mathematics. In Initial Teacher Education (ITE) we expect pre-service teachers to be in the process of developing these understandings necessary to teach mathematics well. Assessing these developing understandings requires that teacher educators first identify the types of knowledge that are critical for the work of mathematics teaching, and then look for evidence of the presence of this knowledge within the pre-service teacher population.

Extensive research has been carried out to identify the types of knowledge required for effective teaching of mathematics resulting in the establishment of a number of different frameworks or models of teacher knowledge categorizing knowledge types. What all these frameworks illustrate is that the knowledge required to teach mathematics effectively is ‘multi-dimensional’ (Hill, Schilling and Ball 2004). This paper explores just two, of the many conceptualizations, of teacher knowledge – those of Shulman (1986) and Ball, Thames and Phelps (2008). The model proposed by Ball et al., has its foundations within Shulman’s work, and was developed within the context of mathematics teaching; these factors influenced the selection of both these models as guiding framework in this study.

Shulman (1986) posits that teachers require three categories of knowledge. These categories are subject-matter knowledge (SMK), pedagogical content knowledge (PCK), and curricular knowledge. Subject matter knowledge refers to *‘the amount and organisation of knowledge per se in the mind of teachers’* (Shulman 1986: 9). According to Ball et al (2008), subject matter knowledge is further categorised into *common* and *specialised* content knowledge. Common content knowledge involves knowledge of the mathematics school curriculum, for example being able to divide fractions. Specialised content knowledge is mathematical knowledge beyond the curriculum – it is the knowledge of mathematics specifically used for teaching.

The second type of teacher knowledge, PCK, focuses more exclusively on knowledge for teaching. Ball et al. categorise pedagogical content knowledge into *knowledge of content and students* (KCS) and *knowledge of content and teaching* (KCT). KCS “combines knowing about students and knowing about mathematics” (Ball et al. 2008). This type of knowledge includes knowledge of common student misconceptions, mathematics that is perceived as interesting or difficult, and common approaches used by children when presented with specific tasks. KCT provides teachers with the understandings required to plan their teaching so that misconceptions are challenged. This planning incorporates attention to the sequencing of instruction to address misconceptions and draws on useful examples to highlight misconceptions. KCT is also necessary to inform the design of a sequence of instruction that provides a trajectory of tasks which build in complexity and at a speed that provides sufficient consolidation of understanding.

Assessing SMK is generally carried out through the use of pen and paper tests. In contrast the assessment of PCK is less straightforward. The construction of assessment items to capture this knowledge is quite difficult, however, another approach is the observation of pre-service teachers as they teach in classrooms. This paper reports on the assessment of pedagogical knowledge of pre-service teachers as they teach, and reflect upon, the classroom teaching of mathematics.

METHODOLOGY

This study was carried out with 20 final year pre-service primary teachers during the concluding semester of their teacher education program. Participants had completed their mathematics education courses (three semesters) and all teaching practice requirements (at junior, middle and senior grades) and self-selected into mathematics education as a cognate area of study.

In this study, pre-service teachers (working in groups of 5-6), and three mathematics educators used *Japanese Lesson Study* (Fernandez & Yoshida, 2004; Lewis, 2002; Lewis & Tsuchida, 1998) to examine the planning and implementation of lessons in classrooms and thus facilitated the design of tools and sequences of instruction to support the development of statistical reasoning with primary children. Participants worked in five groups of 5-6 participants on the design and implementation of a study lesson. This paper examines the work of one group working with senior infant pupils.

The research was conducted over a 12-week semester. While the first phase involved the *research and preparation* of a study lesson i.e. researching the concept of function in order to construct a detailed lesson plan, the *implementation* stage involved one pre-service teacher teaching the lesson in a senior infants classroom while the remainder of the group and the researchers observed and evaluated classroom activity and student learning. Subsequently, following discussion, the original lesson design was modified in line with their observations. The *second implementation* stage involved re-teaching the lesson with a second different class of senior infants and *reflecting* upon observations. The second implementation was videotaped. This cycle concluded with each lesson study group making a presentation of the outcomes of their work to their peers and lecturers at the end of the semester.

This paper reports on the work of one lesson study group- the Senior Infants group, using their mathematics lesson as the unit of analysis. The data illustrate how observation of classroom teaching sheds insights into the PCK demands placed on pre-service teachers when teaching primary level mathematics.

RESULTS

Illustration of ***KCT: Knowledge of Content and Teaching***

KCT was revealed across different lesson components. Knowledge of content and teaching supports teachers when *designing the sequencing of the content of instruction* (Ball et al. 2008). Pre-service teachers carefully designed the sequence of instruction to build in complexity. Initial lesson stages provided opportunities for pupils to develop experience in collecting data (Figure 1). This data collection activity build the knowledge needed for later activities (Figure 2).

Figure 1

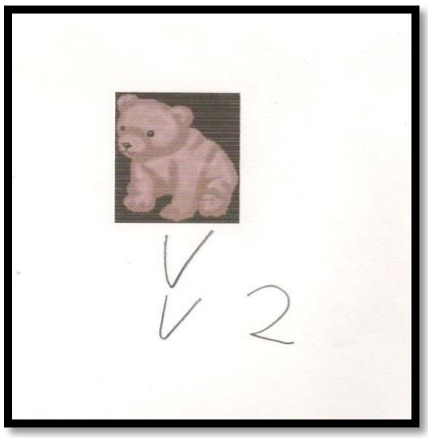

Figure 2

KCT is also revealed through the *selection of models, representations and procedures that support the development of mathematical understandings* (Ball et al. 2008). Pre-service teachers encouraged the construction of concrete graphs of data in an effort to support the developing understandings of data representation on graphs (Figure 3). This indicated their awareness of the difficulties young pupils experience with data abstraction and represented a solution as presented in each data value being represented by a unifix cube. These graphs provided as the precursor to the pictogram constructed by the teacher in conjunction with the class (Figure 4).

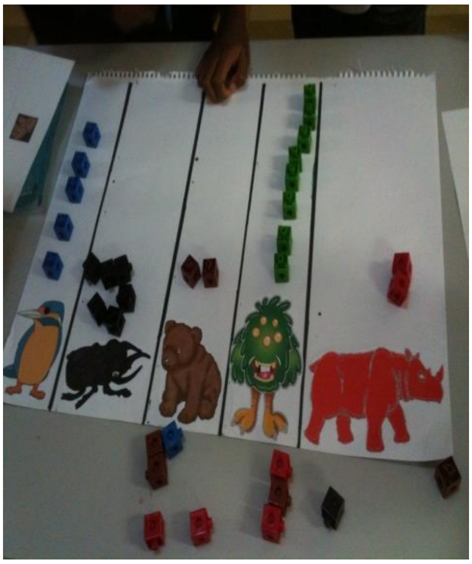

Figure 3 **Figure 4**

Illustration of ***KCS: Knowledge of Content and Students***

The lesson provided evidence of KCS identified in a number of different lesson components. KCS is evidenced in the *ability to select exemplars that motivate and interest students* (Ball et al. 2008). Pre-service teachers wrote a story that engaged and motivated the 6 year old pupils and served as the focus of classroom instruction. Further evidence of KCS was evident in their *ability anticipate student misconceptions when presented with a mathematical task* (Ball et al. 2008). Pre-service teachers were aware of the difficulties children experience with the language of mathematics and had predicted that the use of the word 'more' in the question 'How many more times would red rhino have to come up in the story to beat Green Monster?' may cause confusion. They predicted that the word 'extra' was more accessible to children and used this to supplement meaning to the question (see transcript below). The transcript that follows refer to questions asked based on a pictogram representing the outcome of the data collection (image 4).

Teacher	How many more times would red rhino have to come up in the story to beat Green Monster? This is a really tricky one. How many more times … How many extra times would he have to come to beat Green Monster?
Girls voice	8

Teacher	Let’s see …. Grace.
Grace	9
Teacher	9 more times. So if he came up 9 more times he’d have all these spaces filled and he’d be up to the roof nearly. Wouldn’t he? But he doesn’t have to come up 9 time to
Dara	5
Teacher	.. beat him
Dara	He has to come up 5.
Teacher	So if he had 5 more he’d be right up here.
	So he’d be tied. But we want him to beat Green Monster.
	So, how many times would he have to come up then?
Dara	5
Teacher	I wonder who can solve this one?
Girls voice	11
Teacher	11? It’s not, it’s smaller than 11. He would beat …
	If there was 11 he would definitely beat [Green Monster] but he doesn’t have to come up 11 times. Not even that many. Kerry?
Kerry	23
Teacher	23! Oh we are coming up with very big numbers.
Dara	He would need to come up 6 more .. to beat him
Teacher	Super. Were you going to say that (speaking to another child).
	How do we know 6 more times?
Dara	Because it would be off the chart then
Teacher	It would be off the chart, it would be all the way up to Green Monster and then 1 above him.

Analysis of the transcript also reveals deficits in KCS, specifically around the ability to *interpret the mathematical meaning associated with student responses* (Ball et al. 2008). As can be seen, the pre-service teacher does not realize that the responses of 11 and 23 are correct. These values all satisfy the question criteria. The difficulty itself arose from deficits in KCT pertaining to the ability to *select appropriate mathematical language* (Ball et al. 2008). The intended question pertained to the least number of times that Red Rhino would have to occur to beat Green Monster, hence the only correct answer was 9. However the phrasing of the question did not indicate ‘least’, hence any value greater than or equal to 9 would suffice. Pre-service teachers had not realized this in their lesson design.

CONCLUSION

Lesson study serves as the vehicle wherein participants learn from engaging in and observing teaching; in contrast to traditional pedagogy courses where we just talk about teaching.

While primary teachers are generalist teachers and it is not expected that they are experts in every curricular area, Rowland et al (2009) highlights that teachers are expected to be 'knowledgeable' about their work. Policy makers concur that pupils would learn more mathematics if their teachers knew more mathematics (Kahan et al, 2002). Ball et al (2005: 14) proposes that it is not possible to contemplate improvement of pupils' mathematics achievement without focusing on the nature and effects of teacher practice, that is *'…no curriculum teaches itself…'*.

Lesson study has been found to facilitate pre-service teachers to be a helpful tool in translating the theories presented in traditional lecture-style pedagogy courses to classroom based pedagogical practices (Hourigan and Leavy, 2012; Leavy, McMahon & Hourigan, 2013).

In terms of assessment, while it is common place for instruments (using pen-and-paper assessments) to be developed and administered to gauge student and qualified teachers' knowledge for teaching, these approaches could be considered to be 'narrowly conceived'. It is difficult to ascertain the extent to which performance in an pen-and-paper instrument can provide a conclusive measure of a student teacher's level of preparedness.

In contrast, the nature of Lesson study where there is a particular emphasis on research and reflection provides a vehicle whereby pre-service teachers' knowledge can be examined and developed concurrently within the context of teaching lessons in 'live' classrooms. It facilitates the pre-service teachers themselves to develop the appropriate knowledge as well as making them aware of the shortcomings in their knowledge and the potential for further development. In essence it provides both 'assessment of learning' and 'assessment for learning'.

References

Ball, D.L., H.C. Hill and H. Bass (2005). Knowing Mathematics for Teaching- Who Knows Mathematics Well Enough to Teach Third Grade, and How can we decide? *American Educator*, Fall: 15-46.

Ball, D., Thames, M.H., and G. Phelps, 2008. Content Knowledge for Teaching What Makes It Special? *Journal of Teacher Education, 59*(5), 389-407.

Fernandez, C. & Yoshida, M. (2004). *Lesson Study. A Japanese Approach to Improving Mathematics Teaching and Learning.* Erlbaum: Mahwah, New Jersey.

Hill, H.C., S.G. Schilling. and D.L. Ball, 2004. Developing Measures of Teachers' Mathematics Knowledge for Teaching. *The Elementary School Journal.* 105 (1): 11-30.

Hourigan, M. and Leavy, A. (2012). Primary Mathematics Teacher Education: Teaching at the Heart of Learning in Paul van Kampen, Brien Nolan, Odilla Finlayson, ed.(s), Eilish McLoughlin, chair *ESTABLISH | SMEC 2012 Teaching at the heart of Learning*, Dublin City University, Ireland, 7-9th June 2012, 21-26.

Kahan, J.A., D.A. Cooper and K.A. Bethea, 2002. 'The Role of Mathematics Teachers' Content Knowledge in their Teaching: A Framework for Research Applied to a Study of Student Teachers'. *Journal of Mathematics Teacher Education*, 6: 223-252.

Leavy, A., McMahon, A & Hourigan, M. (2013). Early Algebra: Developing Understanding of the Equals Sign. *Teaching Children Mathematics, 20*(4), 246-252.

Lewis, C. (2002). Lesson *Study: A handbook of teacher-led instructional improvement.* Philadelphia: Research for Better Schools.

Lewis, C. and Tsuchida, I. (1998). A lesson is like a swiftly flowing river: How research lessons improve Japanese education. *American Educator, Winter*, 12-17, 50-52.

Rowland, T., F. Turner, A. Thwaites and P. Huckstep, 2009. *Developing Primary Mathematics Teaching: reflecting on practice with the Knowledge Quartet*. London: Sage.