SOCIOCULTURAL LESSONS FOR REFORM-BASED MATHEMATICS: TRACING PEDAGOGICAL SHIFTS IN A TRANSITION YEAR CLASSROOM

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Recent developments in mathematics education seek change from the traditional exposition and practice methodology to reform methods which link mathematics to the real world and help develop critical thinking and problem-solving skills (NCCA, 2012). This adjustment can be difficult to develop among students whose learning previously has been directed by the traditional (oft textbook- and procedural dominated) approach (e.g. Ross et al, 2002). Transition Year (TY) provides scope, at least 'officially', for this changeover process through its encouragement of a diverse and progressive curriculum (DES, 1994). Our ethnographic-based study, where data was collected over two years (2008 – 2010), aims to gauge how TY students adapted to reform oriented teaching. It demonstrates how TY students engaged collaboratively with mathematical investigations that aided ongoing (formative) assessment that, in turn, enhanced and progressed their learning. This process nurtured growth in confidence as students developed both a stronger sense of 'self' and, ultimately, became independent learners. Throughout the period of research, a number of classroom challenges were encountered by both the mathematics teacher and students as they co-engaged with this change process. Teaching episodes from TY mathematics classes vividly demonstrate how teacher and students struggled (and ultimately succeeded) as active participants of a community of learners. Evidence presented also shows how they co-constructed elements of a mathematics curriculum that had, at its heart, a strong sociocultural design. The teaching and learning effects of the curriculum harmonise with and endorse the pedagogical principles of Project Maths. Moreover, it is shown that the position of TY in providing a forum for such change remains important in paving the way for reform based mathematics in Ireland.

INTRODUCTION AND CONTEXT

The Transition Year (TY) programme which was introduced in 1974 is unique to second level schools in Ireland. As a senior cycle option, it affords students the opportunity to experience different academic subjects, develop new interests, become creatively innovative and engage in vocational preparation (Department of Education and Science, 1994). Its rationale appears to have been based on a desire to move away from a completely exam-orientated system to allow students to be more receptive to new ideas and to develop deeper independence and a higher capacity for conceptual understanding. The Guidelines for TY (ibid.) recommend a balance between academic subjects and a sampling of subjects (e.g. law, media studies, etc.) not generally provided by the school. The core of the programme offers mainly six subject areas: academic subjects, cultural studies, sports, computer studies, work related learning and civic/social studies. All schools offer academic subjects – generally Irish, English, mathematics and a modern European language. TY mathematics offers the opportunity for a more open approach, with a range of methods of presentation and exploration of topic to help stimulate and maintain students' interest. The guidelines for TY advise that with mathematics education:

> "The approach taken ... is as important as the content itself. It should seek to stimulate the interest and enthusiasm of the pupils in identifying problems through practical activities and investigating appropriate ways of solving them. In this way, study can be brought into the realm of everyday life so that the process appears to be more pupil-directed than teacher-directed" (ibid, p.10).

Such an approach is also congruent with the aims of Project Maths which envisages ongoing change to students' learning and assessment in mathematics with a much greater emphasis on conceptual understanding and on the application of learning to other contexts and to the real world. TY mathematics and Project Maths both encourage teachers to reject traditional teaching in favour of more progressive methods which "enable students to have a valid and worthwhile learning experience with emphasis given to developing studying skills and self-directed learning" (ibid, p.3). In teaching TY modules, the guidelines suggest the use of negotiated learning, activity-based learning, group work, project work, visiting speakers and day trips. For mathematics teachers these 'progressive' methods imply, for example: facilitating student-led investigations; supporting students' presentations; using spreadsheets, computer programmes and the internet; engaging with print and mass media; and interacting with people, workplaces and institutions involved with mathematical expertise. The authors of this paper have followed these methods closely while implementing a teaching and learning plan faithful to sociocultural principles. The plan provides for: a variety of activity (such as designing an apartment and recording the cost of living); different forms of action (such as measuring and presenting); and use of a range of tools (such as calculators and the internet). At its heart is questioning and enquiry with students becoming actively engaged in their own learning. The TY curriculum plan facilitates students' co-construction of knowledge, their formulation of new knowledge connections and their linking of mathematics to other subjects and to the real-world.

TY affords students space to mature free from exam-stress so that they may make more informed choices about further education and vocational preparation. It is established that TY students become more learning focussed (Smyth, Byrne and Hannan, 2004) and generally continue to third level which, in turn, enhances their life and employability prospects. In our view, the key pedagogical value of TY is its engagement with more novel ways of learning that enable students to become confident self-reliant individuals as they meet the challenges of Twenty-First Century society.

SOCIOCULTURAL LESSONS FOR MATHEMATICS LEARNING

Sociocultural theory proposes that students learn collaboratively with language playing a key role in the development of their higher mental processes (Vygotsky, 1962, 1978). Here we consider three of its specific conceptual lessons in relation to

TY students' mathematics learning: classroom methodology; assessment; and identity change. In school classrooms, speech, writing, and visual forms of literacy as well as other social tools such as ICT, help mediate social interaction as students work together to develop shared meanings (Wenger, 1998). In keeping with TY aspirations, students are encouraged to "participate in learning strategies which are active and experiential and which help them to develop a range of transferable critical thinking and creative problem-solving skills" (DES, 1994, p.1). Formative assessment plays an important role in this process as it appraises, and evaluates students' performances and uses these profiles to shape and improve their competence (Gibbs, 1999). This complementary assessment process facilitates identity formation leading to a deeper sense of self development (Penuel and Wertsch, 1995). Students are challenged to become active learners, with the teacher no longer being the knowledge-provider but rather a creator of classroom possibilities that stimulate personal and critical forms of mathematical learning (Conway and Artiles, 2005; Van Huizen et al, 2005). Let us now consider the first sociocultural lesson for mathematics.

Classroom Methodology

We sought to develop a mathematics teaching and learning plan inspired by sociocultual learning theory. This plan provided a framework for classroom activities. At the start of class it was important to introduce the learning objective(s) of the activity, giving students a focus and a general approach to new subject knowledge. Thus, a conceptual idea is introduced for exploration – this may be a statement proposing an open investigation such as finding the dimensions of shapes with volume equal to 216 cubic centimetres. As students concentrate on this, their questions and real-world experiences become apparent. By listening to their contributions, the teacher becomes familiar with the students' prior knowledge upon which new understandings will be constructed. Further ideas and suggestions are elicited with such questions as: "What do you think?"; and "Why this?", etc. Sufficient 'wait time' for inner thinking is provided, while students' unique approaches to problem-solving are evaluated and praised. In this way, the teacher models the type of learning attitudes and actions which students are expected to engage with one another, as they work collaboratively. In effect, these 'hidden curriculum' insights present key 'learning to learn' lessons in the mathematics classroom.

Over time, the teacher encourages the growth and development of "a community of practice" (Lave and Wenger, 1991, p.98) in the classroom within which additional characteristics of sociocultural theory are recognisable. Such characteristics include: linking scientific and everyday knowledge; allowing students to put their own words and understandings on the ideas they explore; mediating students' actions by material and symbolic tools; scaffolding – by means of the zone of proximal development (ZPD, see later discussions) and peer groups supports; facilitating individual and collaborative interaction; and group problem-solving. Since "each learner presents a unique profile of abilities, accomplishments, characteristics and needs" (LaCelle-Peterson, 2000, p.39), each class period is different – a position upheld by sociocultural acknowledgement of the power of "situated learning" (Lave and Wenger, 1991, p.30). Within the social and cultural environment of the classroom, both teacher and students work collaboratively together until common knowledge ideally emerges (Gutiérrez et al, 1999). They take ownership of this knowledge and, with time and maturity, become more independent learners.

Assessment

Curriculum and assessment are integral to each another – one guiding objectives, the other seeking assurances that they are being achieved. In facilitating this iterative process, assessment should be a two-way flow, providing "…accurate information with regard to pupil strengths and weaknesses, and [being] formative, so as to facilitate improved pupil performance through effective programme planning and implementation" (Sullivan and Clarke, 1991, p.45). The TY Guidelines (1994, p.4) recommend that: "appropriate modes of assessment should be chosen to complement the variety of approaches used in implementing the programme". Reports, projects, student diary or log book, etc. are among the suggested assessment modes with freedom of type and use advocated. Student involvement is key in facilitating their ownership of learning.

The challenge for the teacher is to integrate methods of assessment which measure students' potential for growth by providing information on "those functions that have not yet matured but are in the process of maturation" (Vygotsky, 1978, p.86). Formative assessment provides feedback for teachers and students on the promotion of effective learning over the course of instruction. When teachers identify how students are progressing and where they have difficulty, they can then make instructional adjustments to promote learning using different approaches. According to an information leaflet produced by the NCCA, Assessment for Learning (AFL) is an appropriate means – being referred to "as formative assessment as its intention is to form, shape or guide the next steps in learning". Student-involvement in the process of assessment facilitates "greater self-awareness and an increased ability to manage and take responsibility for personal learning and performance" (DES, 1994, p.4). Some practices supporting AFL are: classroom questioning, peer and self-assessment and 'comment only' marking (see Black and Wiliam, 1998, 2003; Stiggins, 2002; NCCA, 2005).

Questioning seeks to improve the interactive feedback between students and teacher. By allowing more time for students to answer questions, they become more involved in classroom debates and discussions. Moreover, students are encouraged to explore the validity of their thoughts, to make assumptions, to find convincing arguments to support these assumptions or to find inconsistencies in the thinking of others. Such flexibility in their thinking is important so that they can understand different points of view, and be willing to change their beliefs when further knowledge comes to light. Answers are carefully attended to so that students receive meaningful responses that challenge and enable them to extend their knowledge. The procedure of answering of 'a question with a question' (particularly on the part of the teacher) gives credence also to the importance of problem-posing, as well as problem solving. During this interactive practice, teachers learn more about the thought processes of students, including gaps and misconceptions in their knowledge, and can witness the 'scaffolding' act advancing learning (Bruner, 2006). In 'comment-only' marking, correct work is acknowledged, weaknesses are mutually recognised and advice regarding improvement is forged. Here there is emphasis on learning rather than on performance. With peer- and self-assessment teachers encourage students by providing opportunities to appraise their own and others' work and to review and record their own progress. This gives them valuable insights into their: achievements; understanding of weaknesses in their knowledge; and plans for self-development. With such insights students are well placed to advance their learning and to become more active members of a community of practice.

Identity

Over time, changes in both teacher and pupils may be perceived. The teacher's role becomes imperceptibly modified from being (predominantly) a transmitter of knowledge to (gradually) a facilitator of a sociocultural learning climate that enables students to explore their own learning. This involves considerable personal change (see later discussions). In addition, teachers' professional practices develop to include capacity to: nurture collaborative inquiry; facilitate team work; follow students' thinking; scaffold students' knowledge; and assist students to scaffold each other's knowledge. Overall, classrooms transform gradually to "knowledge-creating communities with questioning and inquiry being central aspects of this process" (Sunderland, 2007, p.40).

In a sociocultural learning climate, students are no longer passive receivers of knowledge; rather they draw on their own prior understandings and actively co-construct new knowledge in more meaningful and collaborative ways (Wenger, 1998). Within a social setting, they look to one another for knowledge, to make decisions, connect mathematics to the real-world, discover information for themselves and establish new knowledge links. While working as creative and constructive problem solvers, their confidence grows and they become more independent learners. Gradually the teacher-student power relationship narrows, as students develop more positive attitudes towards mathematics learning and feel more encouraged to share curriculum choices. To illustrate, students in this study suggested that more student-designed PowerPoint presentations and exhibitions of their work in mathematics be facilitated. It was also recognised that such change would also help them to improve their ICT and public speaking skills. Such 'organic change', so-called because it is not 'forced' on the teacher and students, happens over time at a different 'pace and space'.

A NOTE ON RESEARCH METHODOLOGY

This paper emerges from a wider qualitative research study which took place over two consecutive school years from September 2008 to May 2010. It involved two separate TY classes in a co-educational voluntary secondary school. In the first year of the study there were twenty four students in the class (twelve girls and twelve boys), while in the subsequent year there were sixteen students (twelve girls and four boys). All students had completed Junior Certificate mathematics in the year previous to TY, with thirty six taking higher level and four ordinary level. The main author of this paper was the teacher in the classroom, who had taught many of the students in Junior Cycle and who sought change from traditional to reform teaching approaches. She was supported by advice and encouragement from the co-authors of this paper who acted as mentors offering careful empirical direction and informed conceptual focus. There were ongoing observations of the students by their teacher during their mathematics classes, which consisted of two periods of thirty five or forty minutes and one 'double' of eighty minutes each week. Traditional methods of drill and practice had been previously used to teach mathematics with a strong emphasis on the use of a textbook. Assessment had been in written form, with class tests at the end of a topic or at mid-term and formal end of year examinations in operation.

As the on-going emphasis was on interpreting learning in a social setting rather than testing a particular hypothesis, the research methods used were consistent with the interpretivist paradigm and associative qualitative approaches. These included: classroom observations; field notes; samples of students' work; researcher diary; and focus group interviews. Observation was largely unstructured and although its general focus was clear, there was little clarity initially. Indeed clearer observations emerged over time alongside greater conceptual elucidations of events. Through spending time in the classroom, patterns emerged that greater evidenced theoretical categories. Conversations with students and amongst ourselves also helped to shed light on ongoing and eventual changes. Students were observed during class in relation to changes in behaviour, attitudes, responses, body language and application to tasks. All change was noted as near as possible to their actual occurrence in class. From the beginning of the study, key words, phrases and short quotes were written as accurately and as objectively as possible. Efforts were made to ensure that the note-taking did not interfere with the flow of the lesson or the pupils' actions and reactions. Detailed notes were made later which documented the engagement of students with the knowledge substance, their interactions with each other and the measure of progress of both teacher and students in eliminating the conventional teaching methods of teacher-led exposition and individual student practice. Samples of students' work too were gathered by the teacher to evidence the change (if any) of the students' engagement with reform mathematics. Throughout the project the teacher kept a diary, which became more personal/professional in nature, compared to (arguably) the more objective professional focus of field notes. Here there was opportunity to subjectively reflect on the research, consider changes of direction, generate new ideas, comment on pitfalls, problems, etc.

Students' and parents' views about mathematics learning were also explored by means of semi-structured focus group conversations. Questions were of an open nature, providing a frame of reference for answers, but putting little control on participants to allow for a free flow of information. Students' thoughts were sought on how they thought the teacher expected them to work in class, the best ways they had found to learn and understand mathematics, the renewed classroom arrangements, homework and methods of assessment they found most effective and their views on the mathematics curriculum and its improvement. Parents' opinions were evoked on their own in-school mathematics learning, their expectations of and benefit to their children of TY mathematics, reform-teaching, homework and assessment.

During this study, data obtained by different research methods required specific and inter-related analyses. The qualitative data was continually and eventually categorised with recurring themes being identified that formed a basis for a multi-related coding system. As there was no set method of coding, what was involved was a mutual fitting between data and categories. Some data fitted into more than one category, other data did not fit neatly into any category, while other data created its own category. With the fieldwork and data collection 'officially' concluded, a continuous cycle of reading, interpreting and editing helped to develop the categories, elicit key findings, as well as possible recommendations and issues for further inquiry. The three main themes of sociocultural reform discussed in this paper - classroom methodology, assessment and identity shifts – were evidenced empirically. These are discussed below. Together they harmonise to engage and progress students' interest in mathematics – important at a time when the teaching and learning of mathematics in Irish schools is perceived to be 'causing concern' (Engineers Ireland, 2010).

THE EMPIRICAL STUDY: TRACING SHIFTS IN TEACHING AND LEARNING

This section of the paper describes key pedagogical changes which occurred during the implementation of a mathematics teaching and learning plan that was informed by sociocultural learning principles (Vygotsky, 1962, 1978; Boaler, 1997; Moll, 1990). Aspects of the plan are now described together with related changes in classroom methodology and assessment and emergent student and teacher identities. The change process was not without challenges as traditional pedagogy conceded to a reform-based approach to mathematics that engaged, inter alia, more open problem-solving and group-based activities.

The teaching and learning plan for TY mathematics was not 'set', as it allowed for change owing to constant feedback from pupils, most recent teacher observations, etc. The plan provided for personalised forms of learning, placing the students' abilities, needs and interests at the centre of the educational encounter. There was "more emphasis [placed] on hearing students' voice and encouraging them to be partners in their own learning, rather than spectators" (NCCA, 2008, p.28). In this way, it was possible to engage students in intrinsic mathematical explorations that aim towards 'discovery learning' that is centred on meaningful mathematical knowledge. Students actively co-constructed the plan in a variety of ways: some topics were extended to several class periods when their explorations required more in-depth analysis; other topics were introduced in advance and their views sought on best ways to progress them; while in conversation with the teacher, students made recommendations on the retention, moderation or exclusion of content. The plan was deliberately non exam-driven; rather, it sought to develop concept formation and greater connections of knowledge to the real world. Thus, it was in keeping with the ethos of TY, Project Maths and reform-based teaching.

Material was presented in a variety of different formats including two and three-dimensional representations, photographs, handouts, drawings, games, computer applications and classroom visits. The wider society provided the framework for contextualising knowledge through the use of monetary currency, market products, newspapers, magazines, buildings, people, projects, etc. Such knowledge is more likely to make sense to young people who, in turn, use the tools and artefacts of culture to promote their conceptual development and express themselves more meaningfully (Solomon et al, 2006). These features contribute to the breadth and balance of the programme, facilitating students' coherence of similar knowledge in diverse situations (Gutiérrez and Rogoff, 2003). Meaning is derived from social interactions and the relations students form with others in the learning activity (Wenger, 1998). Sometimes these interactions are with the teacher or more knowledgeable peers, while other times with peers of similar but uneven knowledge repertoires. In this way, students learn to interpret meaning in keeping with the shared understandings of others and are enculturated into a community of practice of mathematics (Brown, 1997).

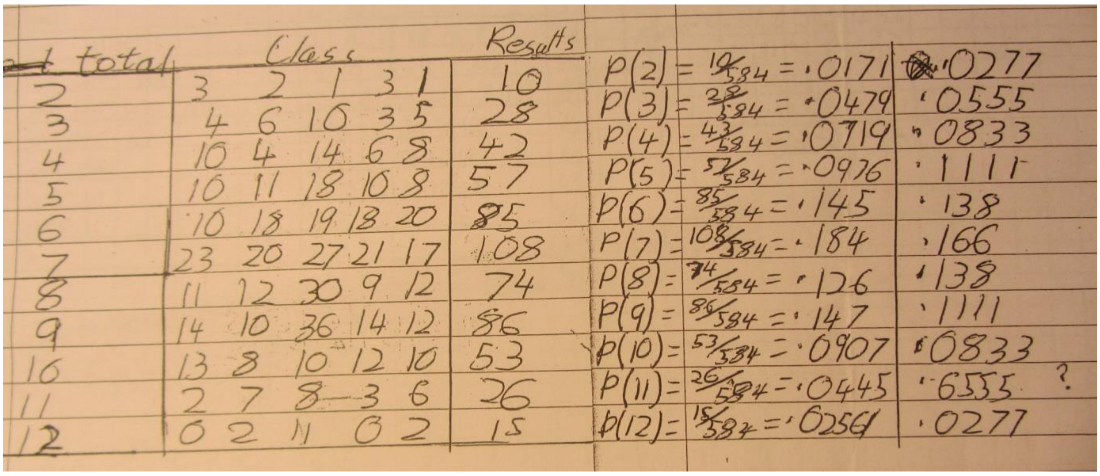
The teaching and learning plan included in this study provides for a variety of activity, ranging from the everyday (e.g. shopping and cooking) to those that are highly specialised (e.g. banking and architecture). One of the mathematical activities involved students designing the layout of a household vegetable garden. This novel project unearthed major gaps in their knowledge such as the names of some vegetables, their appearance, the optimal spacing for growing, tending, etc. To stimulate their thinking the teacher introduced the class to gardening books, magazines, packets of seeds and examples on the internet – classroom resources at odds with traditional classroom practice. Two students with experience of vegetable growing helped answer some of their peers' questions and guided access to further information. The classroom formed a forum for inquiry and exchange, with the project culminating in the cultivation of a garden in co-operation with the agricultural science teacher and her students. This brought home to us the idea that mathematics could be a living discipline for them. It seemed to suggest that the subject could be disassociated from its oft dispiriting image of being remote and lacking context – the latter all too frequently associated with traditional forms of methodology.

In further contributing to ethnomathematics (Gerdes, 1994: Radford, 1997), students were encouraged to bring relevant newspaper and magazine articles to class and to tell their peers briefly what they were about. Over time they became practised at this, presenting interesting items without invitation, thus connecting scientific with everyday concepts (e.g. Vygotsky, 1962). An article on obesity had information on body mass index, while another on sleep showed graphic age comparisons between peoples' sleep patterns. These indicated their growing awareness of mathematics in the world around them. It also gave students an opportunity to source material of interest to them and become providers of knowledge to the class. Conversations with students evidenced their deepening understanding as they connected mathematics to real life. One of the students, Sharon (pseudonyms used throughout), noted:

> "I have good memories of TY maths. I actually really enjoyed them and I liked them. I think I just really understood them really well. It hadn't happened before. It really clicked in TY and all the maths I had learned for Junior Cert. made sense. It had meaning."

Parents confirmed this too with such comments as "They [mathematics] had come alive for her." This contrasted hugely with their own experiences of school mathematics which they identified generally as "a lot of rigour and rules" and "rhyming things off." Learning sociocultural-based mathematics had meaning.

The plan also accommodated a variety of forms of action – sometimes physical with measuring, recording, sowing seed, etc., while at other times, verbal (e.g. presenting projects). More usually both physical and verbal actions occurred, such as during the course of tossing coins for a probability experiment, or finding volumes and surface areas of irregular shapes.



total	Class					Results		
2	3	2	1	3	1	10	P(2) = 10/584 = ·0171	·0277
3	4	6	10	3	5	28	P(3) = 28/584 = ·0479	·0555
4	10	4	14	6	8	42	P(4) = 43/584 = ·0719	·0833
5	10	11	18	10	8	57	P(5) = 57/584 = ·0976	·1111
6	10	18	19	18	20	85	P(6) = 85/584 = ·145	·138
7	23	20	27	21	17	108	P(7) = 108/584 = ·184	·166
8	11	12	30	9	12	74	P(8) = 74/584 = ·126	·138
9	14	10	36	14	12	86	P(9) = 86/584 = ·147	·1111
10	13	8	10	12	10	53	P(10) = 53/584 = ·0907	·0833
11	2	7	8	3	6	26	P(11) = 26/584 = ·0445	·6555 ?
12	0	2	11	0	2	15	P(12) = 15/584 = ·0256	·0277

Figure 1.1: **Sample Space on Outcomes of Tossing Two Dice**

During one of the investigations students completed a sample space on the expected results of totals on tossing two dice. Then they worked in groups replicating the event and recording the outcomes while later they combined the class results to probe how closely they matched with the expected results (Figure 1.1). Here classroom methodology allowed for discussion and physical interaction among students as they learned to align theory with reality and become active learners. It provided scope too for self-assessment as is shown by the question mark alongside the result 0.6555. This evidenced the student's realisation of its variance with the findings in general and of the failure of the numbers to add to 1. Such self-assessment is immediate, allowing the student to examine mitigating factors while the investigation is 'live' with guidance available from peers and/or teacher.

While being engaged in a variety of activity and forms of action students also learn to use tools, both semiotic (e.g. signs and symbols) and physical (e.g. calculators and protractors) which both aid investigations. The use of three-dimensional models, concrete materials and structures helped students apply their classroom information to the real world. A shoe box and cord representing the 'Spider and the Fly'2 problem facilitated analysis of how the spider at one end of the cord might reach the fly at the other (Figure 1.2). In the investigation the fly

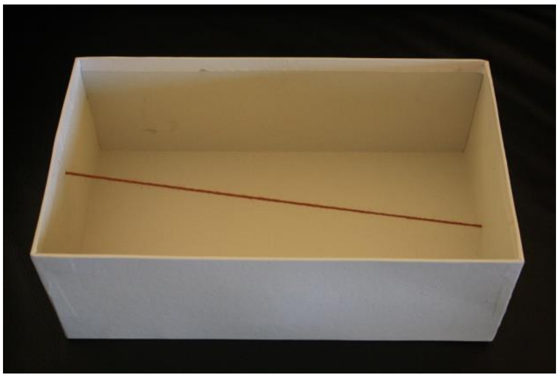


Figure 1.2: **Representation of 'The Spider and the Fly' Problem**

was 1m from the ceiling on the centre of a wall, while the spider was 1m from the ground on the opposite wall. Students envisaged a series of possible routes, committed related steps to paper, outlined and performed accompanying mathematical calculations. Here they learned how to: change perspective from three-dimensional to two-dimensional representation; have different approaches to a problem; and explore various solutions. The use of the shoe box eliminated much of the explanation required in the traditional classroom as students had a concrete representation of the investigation. Furthermore, it assisted in following students' thinking as various options were explored and evaluated. Such use of tools and materials renews the teacher's classroom methodology as he/she seeks to revisit the mathematical representation of enquiries.

Language as a mediational tool plays a major role in students' concept formation (Wertsch, 1985). They listen to explanations in everyday language, ask questions, argue their point of view, talk, think quietly to themselves, etc. Their own idiom and forms of oral expression intertwines with the more formal language of mathematics. 'Official' (often class and adult-based) forms of language can be more easily absorbed when spoken in conjunction with their own speech in the context of their lived experiences. Through language students are enabled "to internalise the world they experience in the living of their lives" (Hasan, 2002, p.113). On one occasion a student stated that she saw no use for the 'Tan of an angle' in trigonometry. A few days previously she had climbed Croagh Patrick with others and in conversation about it, she learned to relate the ease and difficulty of the climb with the incline of the ground (Solomon et al, 2006). "We made good progress at the start as the ground was not so steep but after a while we slowed down as it got very steep", was one of her observations. Together with the class, she utilised rough drawings of a cross-section of the mountain to relate various changes in steepness to the difficulty or ease of the climb. Making the connection between the incline of the mountain to the slope and Tan of its angles facilitated the introduction and understanding of formal mathematical language. Here the exploration of the student's personal knowledge and the forging of connections with scientific knowledge enabled her to make her own subjective meanings. This relates to the sociocultural principle that language is not just a medium for communicating ideas but also fundamental to the formation of ideas and concept development (e.g. Jaworski, 1999).

Students' voluntary attention and active participation are important requirements for the success of learning experiences provided for in the teaching and learning plan. Topics included are designed to encourage students' participation in learning and to prompt them to nominate and carry out their own investigations. Such freedom can help contextualise learning, lead to deeper understandings, and "offer pupils space to learn, mature and develop" (DES, 1994, p.2). One of their investigations involved getting the height of a tree in the school grounds by using a protractor and a single desk to find the angle of elevation of its top, and a tape to measure its full distance. On arriving at the tree, they found the grassed area uneven so they placed the small desk on the concrete path nearby. James, one of the students, objected saying "we will get a better angle nearer the tree, so, we should stay on the grass." Fionnuala, one of his classmates, explained to him that being adjacent to the tree did not matter as each time the table was at another distance, the angle would change to allow for it. She used her arm to point to the top of the tree and aligned it with her line of vision at different distances from the tree to show him. They placed the protractor on the desk holding it erect and tried to read the angle to the top of the tree. Claire suggested that it should be moved to the table's edge and another student might look from its centre to the top of the tree to read the angle – which measured as 58°. They then measured the distance between the tree and the table, and the height of the table, while all participants recorded the information. Their active participation facilitated deeper understanding in an investigation 'outside' the classroom. It was also a new way for them to work together, and they successfully valued and assessed one another's contributions.

During investigations students brought their previous experiences, knowledge, beliefs and perhaps misconceptions to projects and then took responsibility for either learning or re-learning these. In the following example, they worked out possible ways of selecting a committee of two from two men and three women. Conal, a student, thought that the answer was six and he asked Gerry, another student, why it was ten. Gerry wrote A B C D E in his copy to represent the five people. Then he selected them in pairs AB AC AD AE BC BD BE CD CE DE and counted the ten selections. Conal realised that his mistake lay in thinking the committees had to be always a man and a woman, when at times it could be two men or two women. In co-constructing knowledge, students learned both to demonstrate and defend their methods and beliefs, thus contributing to their growing confidence.

By being actively involved in learning students were enabled to include latecomers and recent absentees by explaining current mathematical investigations, while they in turn asked questions and made proposals. As the newcomers became more competent they moved to full participation, indicating that learning is not simply the acquisition of knowledge but rather a process of social participation (Lave and Wenger, 1991). An example of this occurred during work on 'the golden ratio' when Melanie was absent for its introduction but was present the next day when her classmates had pictures and drawings of art works, buildings, etc. to illustrate their new learning. The following exchange occurred between her and two other students, James and Elaine, as they facilitated her active involvement in the ongoing investigation:

Melanie: What's this golden ratio?

James: It is a special rectangular shape that the ancient Greeks used in buildings and art mostly.

Melanie: So, it's a rectangle that only the Greeks used?

James: Well, not just them. Other people use it but the Greeks invented it. It's wider than it's high. See here *(showing her a picture and indicating length and width of the rectangle he has used to illustrate the golden ratio).*

Elaine: The sides are roughly in the proportion 2:3. (*Pause. She sees that Melanie is still confused and draws a rectangle in her copy to show her).* See…if you measure here, 2 cm and here, 3 cm, that's roughly it. *(Pause)* If you divided 3 by 2 you get 1.5 so you can write the ratio 2:3 also as ... 1:1.5.

Melanie: So, it's always 2:3 (*pause).* But, not every rectangle will measure 2 cm by 3 cm.

Elaine: That's true, but you can measure any rectangle and divide the long side by the short side and if you get around 1.5, then it may be the golden ratio. Well, there's a more accurate ratio, which is *(pause as she looks it up)* 1.618. We calculated it yesterday but I find it easy to remember the golden ratio using 2:3.

James: *(Showing her his picture again)* Look…If you measure the length and breadth of this section it fits the golden ratio. The length is 11 cm and the width is 6.5 cm. Now divide 11 by… 6.5 *(using calculator)* … that gives ... 1.692 which is nearly 1.618, so it's very near 1: 1.6

The example also shows Elaine and James as providers of knowledge with Melanie accepting their changed roles by not referring her questions to the teacher. Notice how the more capable peers move from a general explanation of the golden ratio being 'wider than it's high' and of ratio 2:3 to the more accurate ratio of 1.618, and its calculation. Throughout the episode Melanie's threshold of knowledge is carefully and continually assessed by them, as they scaffold her understanding incrementally with further facts while allaying her misapprehensions. The roles of more able peers or adults, together with social and linguistic influences on learning are important factors in Vygotsky's measure of the learner's development relative to instruction.

His description of how the more knowledgeable person helps the less knowledgeable learn and reach higher conceptual levels than he or she would be unable to reach unassisted is known as “the zone of proximal development” (ZPD) (Vygotsky, 1978, p.86). Such instances of scaffolding of knowledge happened frequently as the TY classroom methodology progressed to being more pupil-centred than teacher-centred in orientation.

As students worked in groups co-constructing knowledge, classroom questioning helped assess students’ thresholds of knowledge, incorporate their real-life experiences and guide them to new understandings. During the lead up to the following exchange, tossing a coin 10 times resulted in three heads and seven tails, though they had expected five of each. By exploring this together, they continued in a process of meaning-making and applying their knowledge to betting.

Teacher:	What does a probability of $\frac{3}{10}$ mean?
Student 1:	There are three out of ten chances of something happening.
Teacher:	Would you consider that the event would be unlikely, likely or very unlikely?
Student 1:	Unlikely … though not very unlikely.
Teacher:	Why do you say that?
Student 1:	Well, if it were $\frac{1}{10}$ it would be very unlikely as there is only one chance in ten it might happen, whereas with $\frac{3}{10}$, there are three chances out of ten, which is more likely than $\frac{1}{10}$ ...
Student 2:	It’s like horses. You can bet on them if you think there’s a chance they’ll win a race.
Student 3:	How would you know from the betting that a horse might win?
Student 2:	Short odds like …five to four or… two to one.
Student 2:	But that shouldn’t mean they’ll win. If there are 5 horses in a race then each one should have $\frac{1}{5}$ chance of winning.
Student 3:	Right, but if a horse won his other races or now has Ruby Walsh as jockey he may have a better chance of winning.
Student 2:	Oh…so his chance would improve from $\frac{1}{5}$ to say $\frac{2}{5}$ or higher…

Throughout this meaning-making exchange, classroom questioning is an important tool with both teacher and students questioning and “more knowledgeable peers” answering. It embeds contextualised learning, creating deeper meaning and helps students tease out the symbolic representation of the language used. Formal mathematical language may not make sense as it is not used in their day-to-day activities so it may inhibit their mastery of the subject. Social exchange enables them to see that words (or symbols) hold the key to meaning allowing them to think, abstract, problem-pose and problem-solve. One of the students said later “I hadn’t seen much point in the maths we had in Junior Cert., whereas now I see their value – how they can be used.” Here he indicates that he values meaning-making in mathematics rather than just getting the right answer (Barab and Plucker, 2002).

Conceptual development in mathematics then depends on such meaning-making with students actively participating in the process and connecting knowledge to the real-world.

The process of change from teacher-centred to student-centred mathematics was not straightforward (Conway and Artiles, 2005). Both teacher and students had difficulty in forsaking conventional exposition and practice style classroom teaching and learning and in adopting a suitable framework to guide and support their 'reformed' work. The teacher had to learn to cope with the students' transition of looking to each other for information and knowledge, rather than to her. To assist in this regard she employed classroom formations which enabled her best to draw out their knowledge to the point where either she or the students could progress investigations. Initial difficulty in managing student groups abated, as she recognised when to move in and out of the group in order to facilitate learning. A model she aspired to was Nathan and Knut's (2003) image of the teacher being the "guide on the side" (p.176), one who elicits and engages students' thinking, listens carefully, asks questions, monitors conversations, and decides when to step in and when to step aside. The accompanying classroom noise of students learning socially required adjustment too as she came to accept it as essential to an active community of learners – this ran counter to her 'lived' experiences as a traditional teacher. As these and other challenges were encountered and managed, the teacher experienced greater confidence and a satisfying sense of achievement.

In order to facilitate group work, the teacher arranged the furniture before students' arrival to the classroom. Students then knew at the outset whether they were being asked to collaborate in pairs, in groups, or as a class, thereby helping to focus students on the learning objective(s) for the class period (Gutiérrez et al, 1999). On arrival, and observing say, four chairs arranged around each desk, they would remark "we're in groups today" and were free to sit where they felt comfortable in gender balanced groupings facilitated by the teacher. Students themselves advocated collaborative learning with comments such as "we liked working in groups, because if one of us didn't understand something, another student explained it." During the settling in period to the class, their teacher usually shared information on proposed activities. This was a valuable exercise and an important space for probing new ideas as is illustrated here:

Students: *(As they enter the classroom)* What are we doing today Miss?

Teacher: We're going to look at ways of displaying data. You're familiar with statistics and the methods used there.

Student 1: Oh, you mean bar charts, trend graphs and … I can't remember what else.

Student 2: Are histograms the same as bar charts or are they different?

Student 1: Oh yeah, one has spaces between the bars, the other hasn't. Isn't that right?

Student 3: Then there was one with a curve…what was it? That had a name …cum … something…

Student 4: That's cumulative frequency. I liked that last year. It was shaped like an 'S'.

Student 5: There were others we had in national school that were up and down…to show things like sales of ice-cream.

Student 6: Can we look at them on the Internet?

While listening to initial exchanges among these active learners, the teacher discovered, to some degree, their interests and former knowledge so that she might incorporate them where possible in present or future learning experiences. In this way she learned about their world and found ways to align it to mathematical knowledge, this being especially helpful during the initial period of the study. For her it helped foster change in her personal / professional approach to, and management of, the reform-oriented classroom. Over time, both students and teacher became more adept in dealing with new ideas, and incorporating them into the learning process.

On the learning journey of reform mathematics, students were challenged in many ways, yet they found and welcomed new ways to respond to demands. Their dependence on the teacher's approval of their decisions and actions before initiating steps towards problem-solving was eventually replaced by a sense of critical confidence in their own decisions. Their initial hesitance to adopting different lines of inquiry to progress investigations moderated as they posed and discovered answers to their own questions. Their long-held belief of the teacher or the text book as being the source of all knowledge in the classroom faded as they began to articulate and value their own thinking. With their enjoyment of the social context of learning their difficulty in adapting to group work soon faded. Working independently too, they learned new skills such as compiling articles, surveying, testing hypotheses, accessing, deciphering and presenting information, etc. A satisfying sense of self worth developed through collective knowledge-building and respecting each other's views (Penuel and Wertsch, 1995). Their confidence grew as their contributions were valued by the teacher and as they learned to question the beliefs of others. They developed a sense of responsibility for their own learning by managing related classroom tasks and providing their own curriculum input. It was clear to us that the students formed a community of learners as they themselves became distant from traditional classroom practice.

CONCLUDING REMARKS

It is clear that the teaching and learning plan grounded in sociocultural theory had positive effects on TY students' mathematical applications. It is clear too that students' identities had shifted in accordance with this newfound 'way of knowing'. A new 'way of being', we witnessed, had been fostered. Varied classroom methodology engaged their active participation while reflective questioning helped them draw out information and formative assessment enabled them to recognise gaps in their learning and practical steps towards their remediation. During the evolutionary process of drawing away from the traditional teaching methodology the teacher too had changed. Specifically, she assumed a renewed role as the main organiser and facilitator of a collaborative learning climate. Related identity change in students was apparent in: their engagement and interest in mathematics; their ability to defend their points of view and follow different lines of thinking; and their willingness to correct misconceptions and connect mathematics to the real world. Such advancement of students as critical independent learners is in keeping with the teaching aspirations of Project Maths. Moreover, such change equips students well to a life replete with uncertainties and challenges. Transition Year, so aptly named, remains an important channel for encouraging and facilitating this inevitable change process.

Notes

1 Project Maths is a new mathematics initiative which involves change of syllabus, assessment, teaching and learning of mathematics in Irish second level schools. In particular it aims to promote problem-solving and learning for understanding (see NCCA, 2012).

2 "The Spider and the Fly" problem was created by Henry Ernest Dudeney (1857 – 1930) and can be accessed at www.curiouser.co.uk

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