A Comparison of TIMSS 2011 and PISA 2012 mathematics frameworks and performance for Ireland and Selected countries

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Irish schools recently participated in two major international surveys of mathematics achievement – TIMSS 2011 and PISA 2012. These two surveys originate from different philosophies of mathematics education as reflected in their assessment frameworks and tests. This paper compares the two mathematics frameworks in terms of mathematical content and cognitive processes and in terms of the test results, particularly at the level of performance subscales, in the context of Irish mathematics curricula and results for selected countries including UK and Northern Ireland. Some concerns arising from this analysis are discussed along with recommendations which could inform curriculum review.

INTRODUCTION

In 2011, Ireland participated in the Trends in International Mathematics and Science (TIMSS) Study, which is organised every four years by the International Association for the Evaluation of Educational Achievement (IEA). Whereas TIMSS is offered at both Fourth and Eighth grades (equivalent to Fourth class at primary level in Ireland, and Second year at post-primary level), Ireland participated at Grade 4 only. However, students in Grades 4 and 8 in Ireland will take part in the next TIMSS study in 2015.

In 2012, Ireland participated in the Programme for International Student Assessment (PISA), which is organised by the Organisation for Economic Cooperation and Development (OECD). Unlike TIMSS, PISA uses an age-based sample (students aged 15-years), which cuts across grade levels (Second to Fifth year in Ireland, with a majority of students in Third year).

PISA 2012 was the fifth cycle of PISA in which Irish 15-year olds participated. In the first three cycles (200, 2003, 2006), students in Ireland achieved mean scores on paper-based mathematics that were not significantly different from the corresponding OECD country averages. In 2009, Irish students achieved a mean score that was significantly below the OECD average, suggesting a decline in achievement between 2009 and earlier years. However, in the Irish national report on PISA 2009 (Perkins *et al.*, 2012), it was suggested that low student engagement and factors associated with the scaling of achievement were responsible for the lower performance. In 2012, students in Ireland performed on paper-based mathematics at a level that was significantly higher than the OECD average (Perkins *et al.*, 2013). On a computer-based assessment of mathematics, also administered in 2012, students in Ireland achieved a mean score that was not significantly different from the corresponding OECD average. Thus, students in Ireland did less well on computer-based mathematics in 2012 (the first time computer-based mathematics was included in PISA) than on paper-based mathematics. Mathematics was assessed as a major assessment domain in PISA in 2003 and 2012. This means that PISA included a larger than normal proportion of mathematics items, and that performance on PISA mathematics was reported both in terms of overall performance and of performance on content and process subscales.

A majority of students who participated in PISA 2012 in Ireland were in Third year, and had not studied under the Project Maths curriculum (e.g., Department of Education and Skills, 2015), which was introduced in 24 pilot schools in 2008, and in First and Fifth years in all other schools in 2010. In future PISA cycles, all students in Ireland will have studied the Project Maths curriculum, which places a greater emphasis on understanding of mathematics, and the solving of mathematical problems in real-life contexts, than its predecessor, the pre-2010 Junior Certificate mathematics syllabus (Department of Education, 2000).

DEFINITIONS AND STRUCTURE OF TIMSS AND PISA MATHEMATICS

PISA refers to mathematics as mathematical literacy, and defines it as:

> an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens (OECD, 2013, p. 25).

Clearly, PISA is concerned with students' knowledge of mathematical facts and their ability to use mathematical tools, on the one hand, and with their ability to apply mathematics to real-life situations on the other. Although TIMSS does not provide a direct definition of mathematics, the following statement appears in the TIMSS 2011 assessment framework:

> A prime reasons for having mathematics as a fundamental part of schooling include the increasing awareness that effectiveness as a citizen and success in a workplace are greatly enhanced by knowing and, more important, being able to use mathematics (Mullis *et al*., 2009, p. 19).

Hence, TIMSS is also concerned with mathematics is it relates to future citizenship and participation in adult life. However, the framework also illustrates how TIMSS seeks to establish relationships between the intended curriculum, the implemented curriculum, and the attained curriculum. This implies that TIMSS gathers information about curriculum, and seeks to establish relationships between the TIMSS mathematics test and the curricula of participating countries, as well as between classroom instructional factors and student performance in mathematics. Hence, TIMSS tends to be viewed as a curriculum-based assessment of mathematics, and PISA as an assessment of the mathematics required for future life and education.

The content areas and processes underlying TIMSS mathematics are those typically associated with traditional school-based mathematics. The content areas are Number, Algebra (Grade 8 only), Geometry, and Data & Chance (Table 1). These are quite similar to the content areas found in Project Maths, but quite different from those in PISA. It is particularly noteworthy that PISA does not include an explicit Algebra strand. While it might be assumed that there is a direct correlation between PISA Space & Shape and Geometry (and Trigonometry), this turns out not to be the case. For example, Close (2006) found that none of the PISA 2003 Space & Shape items mapped onto the Geometry or Trigonometry content areas in the pre-2010 Junior Certificate syllabus. This indicates that PISA Space & Shape, which focuses on spatial reasoning and applied problem solving, has a quite different focus from more traditional, theorem-based content area of Geometry and Trigonometry.

Table 1: Content and Processes in TIMSS, PISA and Project Maths Frameworks

	TIMSS	PISA	Project Maths
Content	Number	Change & Relationships	Number
	Algebra*	Shape & Space	Algebra
	Geometry**	Quantity	Geometry & Trig.
	Data & Change***	Uncertainty	Functions
			Stats & Probability
Processes	Knowing	Formulating	Recall ****
	Applying	Employing	Instrumental Understanding
	Reasoning	Interpreting	Relational Understanding
			Solving Problems
			Developing Analytic & Creative Powers
			Appreciation of & Positive Attitudes towards Maths

*Not included at Grade 4; **Geometric Shapes & Measures at Grade 4; ***Data Display in Grade 4. ****Although not stated in the syllabus, underlying processes can be inferred from the statement of aims.

Sources: Mullis et al. (2009), OECD (2013), Department of Education and Skills (2012)

It is perhaps in the area of mathematical processes that TIMSS and PISA frameworks differ most from one another. The processes underpinning TIMSS are Knowing, Applying and Reasoning. Again, these are broadly similar to those underpinning Project Maths (Recall, Instrumental Understanding, Solving Problems, Relational Understanding), but are quite different form PISA, which draws on mathematical modelling as the source of its process categories (OECD, 2013). In this view, the problem solver begins with a problem in a real-world context and *formulates* the problem mathematically, according to the concepts and relationships identified. The problem solver then *employs* mathematical concepts, procedures, facts and tools to arrive at a mathematical solution. This stage typically requires reasoning, manipulation, transformation and computation. Finally, the problem solver *interprets* the mathematical results in terms of the original problem. The use of stages of mathematical modelling as a basis for categorising mathematical processes is new, and its validity has yet to be established. It is unclear at this time whether it can serve as a framework for understanding mathematical thinking, or indeed organising instruction.

RESEARCH COMPARING TIMSS AND PISA MATHEAMTICS

A number of studies have directly compared TIMSS and PISA mathematics, focusing on differences between the frameworks, and differences in performance across countries that have participated in the two studies.

In their comparison of TIMSS and PISA, Ruddock *et al*. (2006) noted that:

- TIMSS emphasises items which require the reproduction of facts or standard algorithms, while PISA focuses on items which demand connections with existing knowledge.
- TIMSS has a larger number of items focusing on Number and Measurement, while PISA items are more evenly spread across their content domains.
- A majority of TIMSS items are multiple choice, while a majority of PISA items are constructed response.
- While TIMSS mathematics items tend to be independent of one another, PISA items include multiple questions based on one stem (problem context)

Ruddock *et al*. note that, while TIMSS and PISA both contain complex language, PISA also has a heavier reading load. They note that the high reading demand of PISA items is often accompanied by a relatively low demand in the mathematics required, reflecting the lower level of mathematics that students can apply in new contexts as opposed to the more familiar ones they encounter in, for example, TIMSS. Wu (2009) also acknowledges that there is a considerable amount of reading in PISA, compared with TIMSS, and speculates that, in countries where reading achievement is relatively higher, students may be exposed to an environment which supports the use of mathematics problem-solving skills in everyday life.

Performance on TIMSS and PISA can be compared at country and item levels. Figure 1 provides a comparison between countries that participated in TIMSS 2011 at Grade 8 and in PISA 2012 (Ireland is not included as it did not take part in TIMSS 2011 at Grade 8). The figure shows that Asian countries Singapore, Hong Kong-China, Chinese-Taipei, Korea and Japan are the five highest-ranking countries in both assessments. On the other hand, a number of non-Asian countries, such as the Russian Federation, the United States, Lithuania and Hungary, perform better on TIMSS than on PISA, while countries such as New Zealand and Norway do marginally better on PISA. And, finally, a number of countries perform at about the same level in both studies, including Australia, Slovenia and Turkey. The country-level correlation between mean scores on TIMSS 2011 (Grade 8) and PISA mathematics is 0.93.

Wu (2009) compared the performance of students in selected Asian (Hong-Kong, Japan, Korea) and Western countries (Australia, England, United States) on specific TIMSS Grade 8 and PISA items. She noted that, at individual item level, Western countries may be at an advantage in PISA, where more items are embedded in real life contexts. Related to this, she argues that Western students may approach PISA problems using a practical, common-sense approach, compared with students in Asian countries, who may adopt a more theoretical stance.

The foregoing will be of interest in terms of looking ahead to the performance of students in Ireland on TIMSS 2015. Second-year students in TIMSS 2015 in Ireland will have studied the Project Maths syllabus, with its emphasis on mathematical understanding and on solving mathematical problems embedded in real-life contexts, which one might expect could convey an advantage on PISA-style items. On the other hand, they will also have studied more traditional mathematics content in Project Maths, including a theorem-based or synthetic approach to Geometry, which might be conductive to doing well on TIMSS.

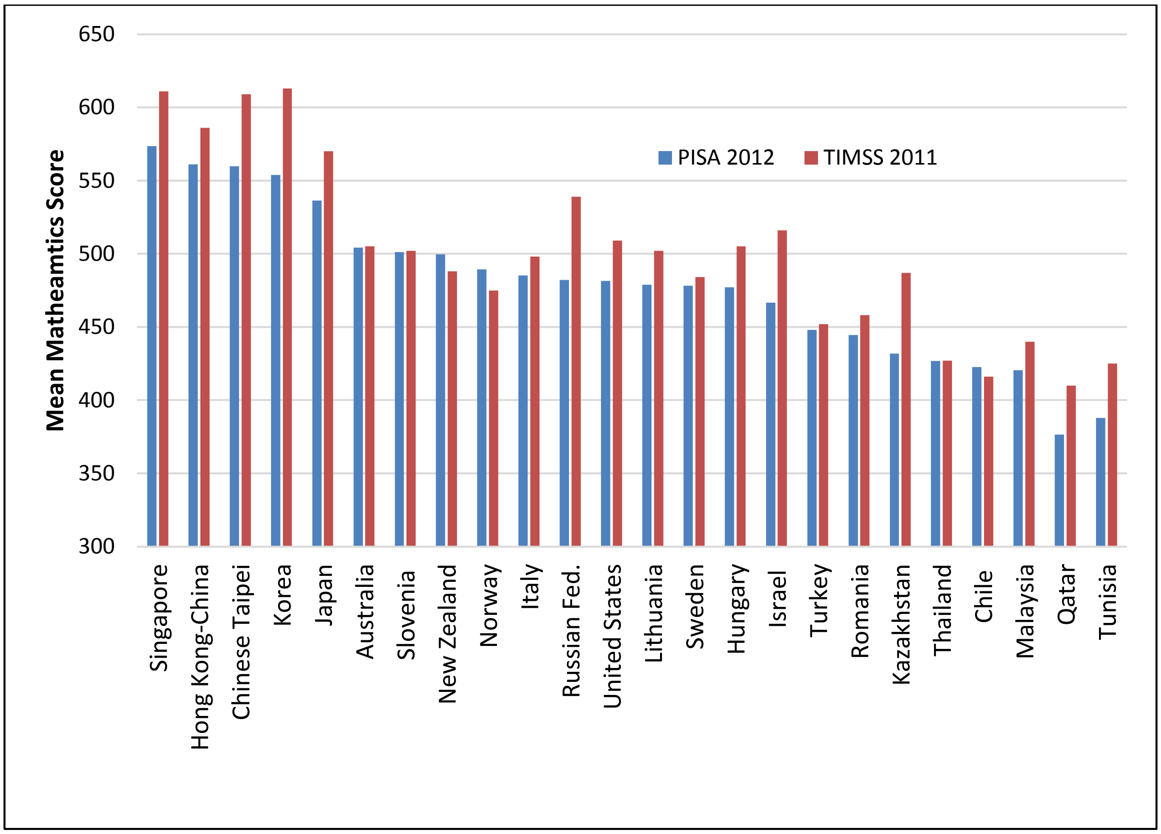


Figure 1: Comparison of Country-Level Performance on TIMSS 2011 (Grade 8) and PISA 2012

IRELAND IN TIMSS AND PISA

Although data for Ireland on TIMSS 2011 Grade 8 mathematics are not available, we can draw some broad conclusions about the performance of students in Ireland based on their performance on TIMSS 2011 Grade 4 mathematics and PISA 2012 mathematics at age 15, relative to other countries in both assessments. Figure 2 shows the relative rankings of countries that participated in both assessments. The figure again shows that Asian

Countries – Singapore, Hong-Kong China, Chinese Taipei, Korea and Japan – were the highest-ranking countries in both studies, suggesting that the foundations for strong performance at age 15 on PISA mathematics may be established by the middle of primary schooling. It is also noteworthy that Ireland performed at about the same level on PISA (11th) as on TIMSS (13th), among countries in both studies. However, a number of countries show quite different rankings across the two studies, including Poland (8th in PISA, 29th in TIMSS), the Russian Federation (7th in TIMSS, 20th in PISA), and the United States (8th in TIMSS, 22nd in PISA).

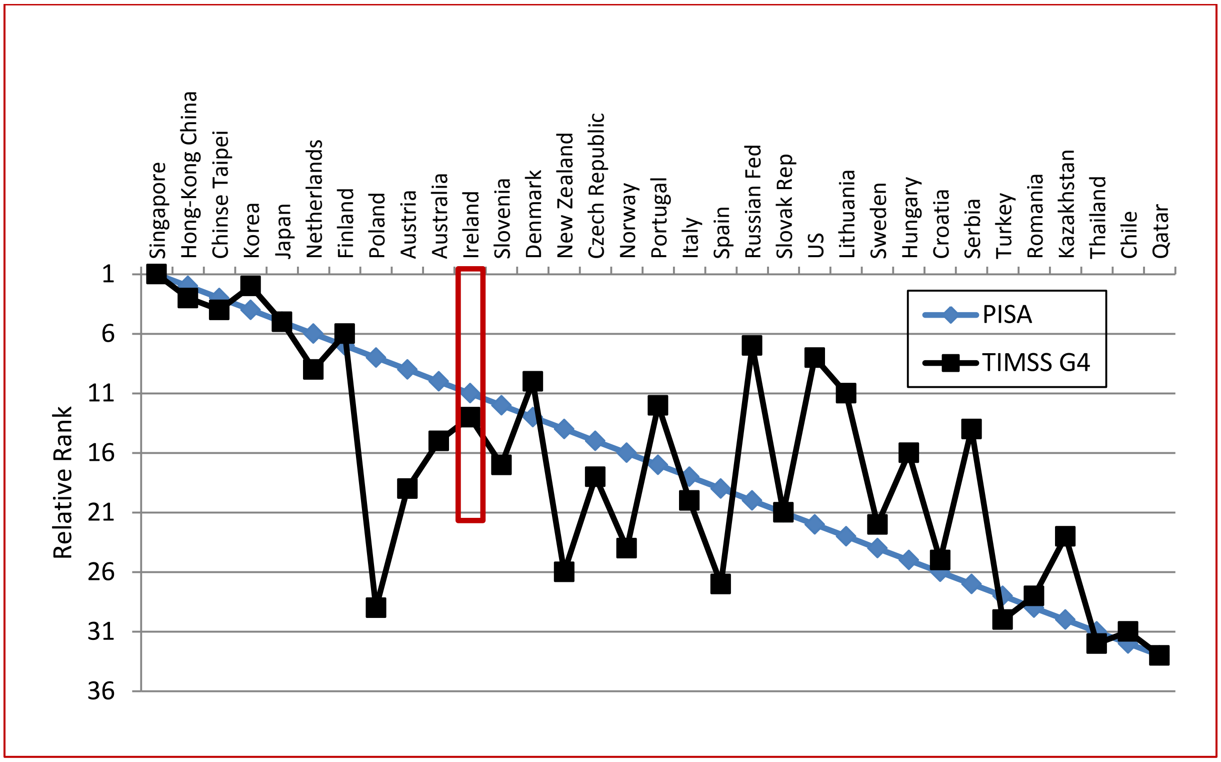


Figure 2:Relative Rankings of Countries in TIMSS 2011 (Grade 4) and PISA 2012

In addition to data on overall performance in mathematics, TIMSS and PISA provide data on performance by content area and process. In TIMSS 2011 (Grade 4), students in Ireland performed at a level that was significantly above their overall score on Number (difference = + 4 scale points), and at a level that was significantly below their overall score on Geometric Shapes & Measures (-7) and Data Display (-4) (Eivers & Clerkin, 2012; Mullis *et al*., 2012)). On the TIMSS process skills, students in Ireland achieved at a level that was significantly higher than their overall score on Knowing (+12), and at a level that was significantly below this on Reasoning (-18). There was no difference between performance on Applying and overall performance. Hence, TIMSS suggests a relative weakness on Geometric Shapes & Measures, on Data Display, and on Applying.

The overall performance of students in England (542) and Northern Ireland (562) was significantly higher than in Ireland (527) on TIMSS mathematics. Like students in Ireland, students Northern Ireland did better on Number (+4), and less well on Data Display (-8), compared with their overall performance. They also did better on the

Knowing process (+17), and less well on Reasoning (-25), than on the test as a whole, with no difference on Applying. Students in England did less well on Number (-3), and better on Data Display (+7) than on the test as a whole. They also did better on Knowing (+10), and less well on Reasoning (-11), with no difference on Applying. While most TIMSS 211 countries tended to do less well on Reasoning than on the test as a whole, students Australia, Finland and Korea performed at the same level on Reasoning as on the test as a whole.

Analyses of TIMSS data at the individual item level (Close, 2013) suggest that there are gaps in the mathematical knowledge of students in Ireland. For example, on an item requiring students to select the length of a piece of string (Figure 3), just 16% of students in Ireland provided a correct response, compared with an international average of 28%. Given the relatively strong emphasis on estimating length in the Primary School Mathematics Curriculum in Fourth class (NCCA, 1999, one would have expected a stronger response from students in Ireland. Other items on which students in TIMSS 2011 in Ireland did not do very well included one involving rotation as a geometric transformation (which was on the pre-1999 mathematics curriculum, but no longer features), one involving identification of the factors of 12, one involving basic multiplication (23 X 19) and one involving distance and time (speed).

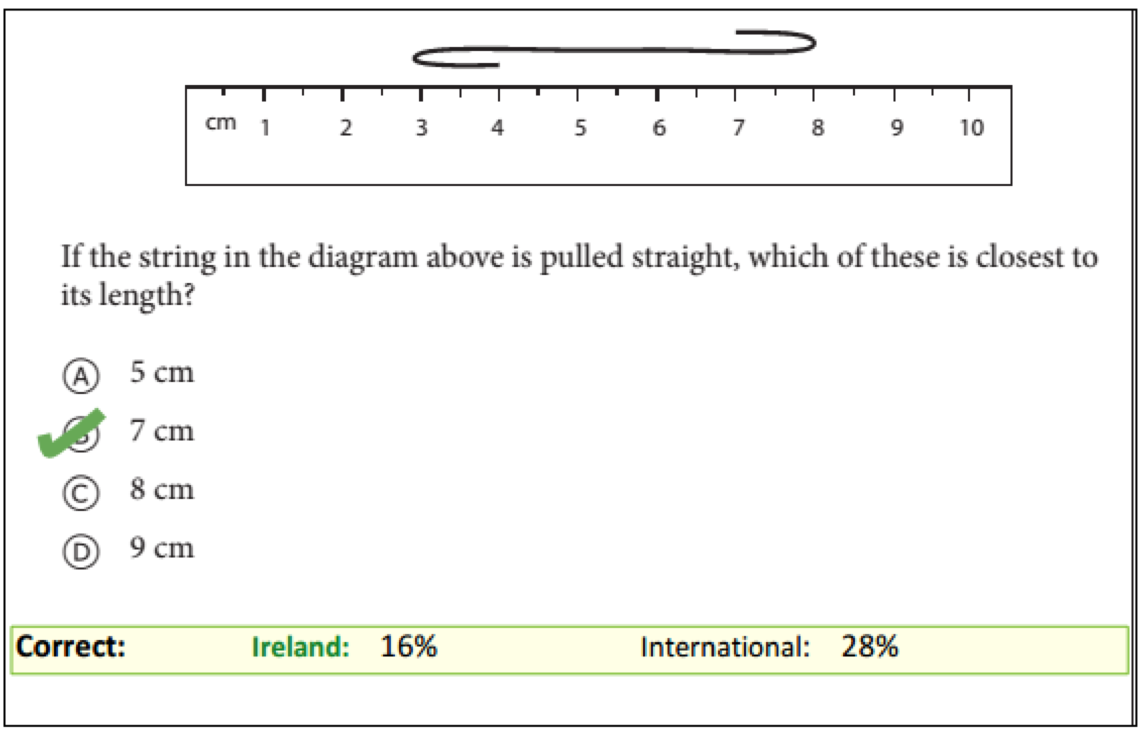


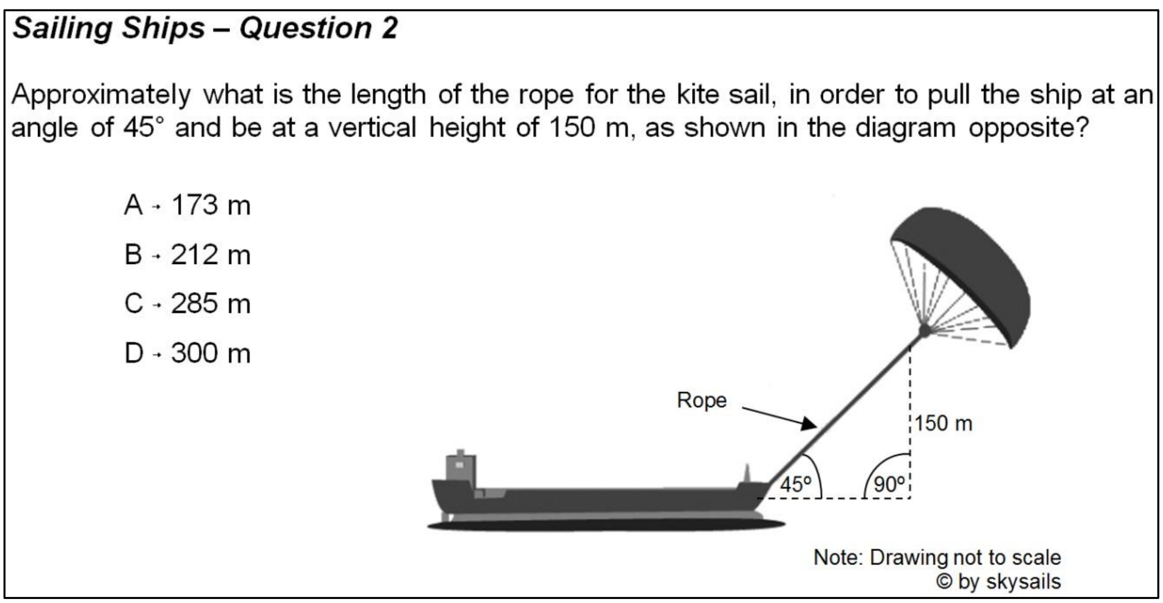
Figure 3:Sample TIMSS 2011 (Grade 4) Item

On PISA 2012, students in Ireland achieved mean scores that were above the corresponding OECD averages on three mathematics content areas – Change & Relationships, Quantity, and Uncertainty & Data. On the fourth – Space & Shape –

students in Ireland achieved a mean score that was significantly below the OECD average. In relative terms, performance in Ireland was strongest on Uncertainty & Data (mean score = 509), and weakest on Space & Shape (478). Female students in Ireland performed particularly poorly on Space & Shape. On the PISA process subscales, students in Ireland achieved mean scores that were significantly above the corresponding OECD averages on Employing and Interpreting. Performance on Formulating was not significantly different from the corresponding OECD average.

PISA 2012 students in Ireland achieved an overall mean mathematics score (502) that was not significantly different from that of the UK as a whole (494). Like Ireland, students in the United Kingdom had a mean score on Space & Shape that was significantly below the corresponding OECD average. Students in Northern Ireland had an overall mathematics mean score (487) that was significantly below the mean score for Ireland and the OECD average. Students in Northern Ireland achieved a mean score on Space & Shape (463) that was below the corresponding OECD average, and a mean score on Data & Change that was not significantly different.

Figure 4 provides an example of a PISA Space & Shape item, where students are required to apply the Pythagorean theorem in a real geometric context. Students in Ireland achieved a mean percent correct score of 48%, compared with an OECD average of 50%. Given that the Pythagorean theorem features strongly on both the pre-2010 and Project Math syllabi, one would have expected students in Ireland to have done better.


Sailing Ships – Question 2

Approximately what is the length of the rope for the kite sail, in order to pull the ship at an angle of 45° and be at a vertical height of 150 m, as shown in the diagram opposite?

A - 173 m

B - 212 m

C - 285 m

D - 300 m

Figure 4: Sample PISA 2012 Mathematics Item

CONCLUSION

International studies of mathematics achievement can provide useful information about overall performance, as well as performance on mathematics content areas and processes. Data for Irish students from TIMSS 2011 (Fourth grade) and PISA 2012 (15-year olds) will soon be augmented with data from TIMSS 2015 (Fourth and Eighth grades), and from PISA 2015, where the mathematics test will be offered on computer only for the first time.

While Ireland performed above the OECD average on PISA paper-based mathematics for the first time in 2012, performance among students in Fourth grade on TIMSS 2011 was weaker, with students in Ireland lagging well behind a cluster of Asian countries, and several European countries, including Northern Ireland, Finland, England, the Netherlands, and Denmark.

Students in Ireland who participated in TIMSS 2011 mathematics showed a relative weakness on Geometric Shapes & Measures, and, to a lesser extent, on Data Display. Performance was also weak on the Reasoning process subscale. In PISA 2011, students in Ireland performed well on three of the four content areas assessed. The exception was Space & Shape, which covers spatial reasoning as well as more general mathematical problem solving.

Although the implementation of Project Maths, which began in all schools in September 2010[1], can be expected to bring about some improvements in all aspects of mathematics, it is unlikely that overall performance on PISA can improve without allocating more specific attention to Space & Shape, including a consideration of the cross-over between PISA Space & Shape and the Project Maths syllabus. There may also be value in considering the extent to which the approaches to other aspects of mathematics in Project Maths (e.g., Algebra) are consistent with PISA Space & Shape. Finally, there may be value in providing short courses on spatial reasoning (e.g., Uttal *et al.*, 2013). A decline on PISA Data & Chance beteen 2003 and 2012 is also a matter of concern.

The relatively poor performance of students in Ireland on the Geometry & Measurement and Data Display content areas, and on the Reasoning process suggests that plans to revise the Primary School Mathematic curriculum (DES, 2011) should proceed without delay, and better bridges should be established between mathematics at the upper-end of primary schooling, and at the lower end of post-primary schooling (e.g., NCCA, n.d.). The recent publication of a *Shape and Space Manual* for primary schools by the Professional Development Support Service (2013) should also point to a broader range of activities for developing a sense of Space and Shape.

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[1] Implementation of Project Maths began in 24 initial schools in September, 2010.

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