A study of the opportunities for creative reasoning in undergraduate Calculus courses

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Our Research Questions

- This project concerns the reasoning expected of first year undergraduate students in Mathematics modules.

- Our research questions are:

- What kind of reasoning is expected of students in first year calculus modules in our universities?

- Is there a difference between the type of reasoning expected in specialist and non-specialist courses?

- Is there a difference between the type of reasoning expected on assignments and examinations?

Reasoning

- Two of the main aims of mathematics modules at university are to develop students' understanding of the subject and to develop their thinking skills.
- There are many different definitions of 'understanding', and it is difficult to study how a student develops understanding.
- This project focuses on how thinking skills are developed. It is often said that the study of mathematics promotes the development of thinking skills and reasoning.
- By *reasoning* we mean 'the line of thought adopted to produce assertions and reach conclusions in task-solving' (Lithner 2008).
- This definition includes both high and low quality arguments and is not restricted to formal proofs.

Why is reasoning important?

- There is a focus currently on how best to foster critical thinking skills in undergraduate students (HEA & NCCA 2011).
- Studies have shown (Boesen et al. 2010) that:
- the types of tasks assigned to students can affect their learning;
- that the use of tasks with lower levels of cognitive demand leads to rote-learning by students and a consequent inability to solve unfamiliar problems or to transfer mathematical knowledge to other areas competently and appropriately.
- It is therefore important to investigate whether first year students in our mathematics modules are given sufficient opportunities to develop their reasoning and thinking skills.

What is known about reasoning in undergraduate modules?

- Some commentators assert that students ‘can pass courses via mimicry and symbol manipulation’ (Fukawa-Connelly 2005, p 33), or learn a large number of standardised procedures in their mathematics courses but not the ‘working methodology of the mathematician’ (Dreyfus 1991 p. 28) and thus may not develop conceptual understanding or problem-solving skills.

- Studies in the UK (Pointon and Sangwin 2003) and Sweden (Bergqvist 2007) found that the majority of tasks in Calculus courses could be solved by using routine procedures and did not require higher order thinking skills.

- One of our aims is to investigate if this is the case in Ireland also.

Lithner's Reasoning Framework

- Lithner (2008) distinguishes between imitative and creative reasoning.

- Imitative reasoning has two main types: memorised (MR) and algorithmic (AR).

- Memorised reasoning is characterised by

1. The strategy choice is founded on recalling a complete answer.
2. The strategy implementation consists only of writing it down. (Lithner 2008, p258)

- Algorithmic reasoning is characterised by

1. The strategy choice is to recall a solution algorithm.
2. The remaining reasoning parts of the strategy implementation are trivial for the reasoner, only a careless mistake can prevent an answer from being reached. (Lithner 2008, p259)

Creative Reasoning

- Lithner calls a reasoning sequence creative if it has the following three properties:

1. Novelty. A new (to the reasoner) reasoning sequence is created, or a forgotten one is re-created.

2. Plausibility. There are arguments supporting the strategy choice and/or strategy implementation motivating why the conclusions are true or plausible.

3. Mathematical foundation. The arguments are anchored in intrinsic mathematical properties of the components involved in the reasoning. (Lithner 2008, p 266).

Our plan

- At the moment we are studying two modules at DCU and two at NUI Maynooth.
- The modules are: a Business Mathematics module; two different Calculus modules for Science students; a Calculus module for Pure Mathematics students.
- The data in this project consists of : lecture notes, textbooks, assignments, examination questions.
- The data analysis of each module is currently being carried out by two independent researchers from the research team who do not work in the home university of the module.

Analysis

- We began by classifying exercises from a chapter of a Calculus textbook, in order to get some practice on ‘neutral’ material and to agree on our methods.
- We have developed a procedure for the classification (in line with Lithner 2008 and Berqvist 2007) as well as a template to record the deliberations on each task.
- We first construct a solution to the task and this is then compared to the course notes and textbook examples.
- Using Lithner’s framework, the researchers decide whether the task could be solved using imitative reasoning or whether creative reasoning is needed.

Classification procedure

- Step 1. Analysis of the **tasks**
- A solution (we agreed to sketch the 'lowest common denominator' in sub-procedures).
- Step 2. Analysis of the **text and previous questions**
- Occurrences in examples and previous exercises;
- Occurrences in text.
- Step 3. Argument and conclusion

- **Local Creative Reasoning or Global Creative Reasoning?**
- LCR: if one sub-procedure is new,
- GCR: - if two or more sub-procedures are new or
- - if a proof aspect is the novel element or
- - if transfer is the novel element.

Example - *Write down the solutions to the following equation: (x-2)(x+1)(4-x)=0.*

- *Task Analysis*: Using the factor method, since *(x-2)(x+1)(4-x)=0,* we conclude that *x=2,-1,4 are the solutions.*

- *Text Analysis*. *Occurrences in the notes*: The factor method and an example can be found on page 19, but there is no example with three factors.
- *Occurrences in the text*: The factor method is given on pages 134 and 135 of the book and used in examples on page 135; however the examples do not cover the case of three factors.

- *Argument and conclusion*:
- This is a Creative Reasoning (CR) task, specifically it is a Local Creative Reasoning (LCR) task. The students can use the factor method algorithm from the notes and the textbook however they need to modify it to handle the three factors.

Results

- So far we have classified the tasks from the Business Mathematics course and one of the Calculus for Science modules.
- The inter-rater reliability for both courses was high; it was over 90% in both modules.
- Both modules had regular assignments which were submitted and counted towards the continuous assessment portion of the module grade.
- They both had practice or tutorial questions, and the Science module also had optional questions.
- The assignment, practice, optional and examination questions were analysed.

Results – Business Mathematics

Reasoning Type	Frequency	Percentage
Imitative Reasoning	**213**	**78.3**
Memorized Reasoning	0	0
Algorithmic Reasoning	213	78.3
Creative Reasoning	**59**	**21.7**
Local Creative Reasoning	37	13.6
Global Creative Reasoning	22	8.1

Results – Business Mathematics

	Practice	Submitted	Examination
AR	93 (62.8%)	100 (98%)	20 (90.9%)
LCR	33 (22.3%)	2 (2%)	2 (9.1%)
GCR	22 (14.9%)	0 (0%)	0 (0%)

Results – Business Mathematics

- We saw that 21.7% of the tasks in this course were CR tasks.
- When we consider the different types of questions, we see that most of these tasks were practice questions and the assessment questions contained few CR tasks.
- 37% of Practice questions were classified as CR, while 2% of Submitted questions and 9% of exam questions were in this category.

Results – Calculus for Science

Reasoning Type	Frequency	Percentage
Imitative Reasoning	**153**	**70.8**
Memorized Reasoning	0	0
Algorithmic Reasoning	153	70.8
Creative Reasoning	**59**	**29.2**
Local Creative Reasoning	33	15.3
Global Creative Reasoning	30	13.9

Results – Calculus for Science

	Practice	Submitted	Optional	Examination
AR	97 (83.6%)	40 (72.7%)	1 (3.6%)	15 (88.2%)
LCR	15 (12.9%)	11 (20%)	5 (17.9%)	2 (11.8%)
GCR	4 (3.4%)	4 (7.3%)	22 (78.6%)	0 (0%)

Results – Calculus for Science

- Almost 30% of tasks in this course were classified as CR tasks.

- The submitted homework contained a relatively high percentage of CR tasks (27.3%).

- The examination had fewer CR tasks (11.8%) – all LCR.

- In this course, there was a difference between the types of reasoning on the practice tasks and the optional questions.

Conclusion

- The two courses contained some CR tasks (29% in Science and 22% in Business). The differences were not significant.

- Most of the CR tasks came from practice or optional questions, with fewer opportunities for creative reasoning on examinations.

- The Swedish examinations contained more CR tasks (31%) than the Irish ones.

- It can be difficult for the researchers to decide between AR and LCR, and between LCR and GCR.

Future Work

- Analyse the tasks from the remaining two courses.
- Compare all four courses to look for any similarities or differences between different types of modules.
- Possibly modify the framework to take different levels of Algorithmic Reasoning into account and to make it easier to decide between AR and LCR.

References

- Bergqvist, E. (2007). Types of reasoning required in university exams in mathematics, *Journal of Mathematical Behavior*, 26, 348-370.
- Boesen, J., Lithner, J, & Palm, T. (2010). The relation between types of assessment tasks and the mathematical reasoning students use. *Educational Studies in Mathematics*, 75, 89-105.
- Dreyfus, T. (1991). Advanced Mathematical Thinking Processes, in Tall D. (ed) *Advanced Mathematical Thinking*. Kluwer Academic Publishers.
- Fukawa-Connelly, T. (2005). Thoughts on learning advanced mathematics. *For the Learning of Mathematics*, 25, 33-35.
- HEA and NCCA (2011). *From transaction to transmission: Outcome of the conference on transition from second to third-level education in Ireland*. Dublin.
- Lithner, J. (2008). A research framework for creative and imitative reasoning. *Educational Studies in Mathematics, 67*, 255-276.
- Pointon, A., and C. Sangwin (2003). An analysis of undergraduate core material in the light of hand-held computer algebra systems, *International Journal of Mathematical Education in Science and Technology*, 34(5), 671-686.