Intervention-based research in (mathematics) teacher education*

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1

Teachers' subject knowledge: importance

- Effective teaching in the different subject areas requires that teachers have a strong subject knowledge (e.g., Ball, Lubienski, & Mewborn, 2001; Ball, Thames, & Phelps, 2008; Kilpatrick, Swafford, & Findell, 2001; Ma, 1999; Shulman, 1986)

  …in addition of course to other kinds of knowledge
  - knowledge of students
  - knowledge of teaching or pedagogy
  - knowledge of the curriculum…
- What might mathematics subject knowledge include?
  - "Knowledge of mathematical facts, concepts, procedures, and the relationships between them; knowledge of the ways that mathematical ideas can be represented; and knowledge of mathematics as a discipline – in particular, how mathematical knowledge is produced, the nature of discourse in mathematics [e.g., problem solving], and the norms and standards of evidence that guide argument and proof" (Kilpatrick et al., 2001, p. 371)

  …as well as subject-related beliefs and underpin productive use of these types of subject knowledge (e.g., Ponte & Chapman, 2008)

2

Teachers' subject knowledge: limitations

- A common theme identified in reviews of research on teachers' mathematics subject knowledge (e.g., Ball et al., 2001; Ponte & Chapman, 2008) is that there are serious issues with (preservice) teachers' mathematics knowledge that teacher education programs ought to address
  - Issues tend to be more serious (1) for (preservice) elementary teachers than for secondary mathematics teachers; (2) in relation to knowledge of mathematics as a discipline (e.g., problem solving, proof) than to knowledge of more standard curricular topics (e.g., number and operations)
- Reflection on the state of research in this area:
  - Research has focused more on documenting important and persistent problems of (preservice) teachers' subject knowledge (including subject-related beliefs) and less on designing interventions to find promising solutions to these problems that can practically be used in teacher education programs

The issue of an intervention's duration

- The few promising interventions available tend to have long duration
  - Example: an intervention, comprising 3 semester-long classes in an undergraduate mathematics program for preservice elementary teachers based on the history of mathematics, was found to lead to significant improvement of participants' attitudes towards mathematics (Philippou & Christou, 1998)
  - Challenges with promising interventions of long duration:
    (1) lots of potentially-critical factors (What key factors have contributed to the interventions' success?)

    (2) teacher educators need to do major changes to their current courses to accommodate the demands of such interventions
  - Possible risk: "partial" adoption or poor fidelity of implementation of these interventions which can jeopardize their theoretically essential components and compromise their impact (Yeager & Walton, 2011)

Key question of this presentation

Can we design interventions of **short duration** in teacher education to help alleviate **important and persistent problems** of preservice teachers' subject knowledge (including subject-related beliefs)?

- Significance: (1) Effective interventions of **short duration** would make it easier for researchers to tease out their theoretically essential components and more practicable for teacher educators to incorporate them into existing courses thus allowing also better chances of good fidelity of implementation
  (2) Finding ways to address **important and persistent problems** of teachers' subject knowledge can offer high leverage for improving students' learning as teachers' own difficulties with the subject matter impact the learning experiences they offer to their students
- Specific context: elementary teacher education; mathematics

5

A 4-year design experiment in an undergraduate mathematics course for preservice elementary teachers

6

The study

- Context: an American undergraduate mathematics course for students majoring in different fields of study and who aspired to join the masters-level, elementary teacher education program
  (1 semester, 3hrs per week; no other subject-knowledge course in the program so it covered a wide range of topics; a mathematics pedagogy course followed)
- Overarching aim: to develop theory-based and empirically-tested interventions to promote important but difficult-to-achieve learning goals related to preservice teachers' mathematics subject knowledge and subject-related beliefs
- Design experiment methodology (e.g., Cobb et al., 2003)
  - 5 research cycles of implementation, analysis, and refinement of the interventions over a 4-year period
  - One of the researchers was also the instructor (teacher educator)
  - Data during Cycle 5: a pre/post-test of subject knowledge; a pre/post-beliefs survey; post-course individual interviews; videos of all sessions; fieldnotes; copies of all work produced during sessions

7

Three general characteristics of our instructional approach in the course

(1) "Ambitious instruction" (e.g., Franke et al., 2007; Kazemi et al., 2009)

- Focus on deep learning: preservice teachers engaged in problem solving and reasoning in the context of rich tasks
- Dynamic pattern of classroom interaction: teacher educator was expected to be responsive to preservice teachers' ideas; preservice teachers were expected to actively engage in discussions, share/respond to/build on ideas
- Major challenge: high demands on the teacher educator for in-the-moment decision making

8

Three general characteristics of our instructional approach in the course

(2) "Instructional engineering"*

- Description: A deliberately designed task (or task sequence) and a well-developed implementation plan to support 'unforced' learner progression towards a fine-tuned, highly pre-specified learning trajectory to accomplish an important learning goal
- What it looked like in practice: While in the eyes of an outside observer the instructor's responsiveness to classroom participants' contributions during the lesson could appear to place high demands on him for in-the-moment decision making, in reality, the implementation unfolded quite predictably, on the basis of the well-developed plan
- Development of instructional engineerings: conceived, empirically fine-tuned, and theorized over the cycles of the design experiment

* Stylianides, G. J., & Stylianides, A. J. (2014). The role of instructional engineering in reducing the uncertainties of ambitious teaching. *Cognition and Instruction*, *32*, 374-415.

Three general characteristics of our instructional approach in the course

(3) "Mathematics for teaching as a form of applied mathematics"*

- Description: Given that the students on the course were adults who specifically aimed to become teachers of mathematics, the development of their mathematics subject knowledge could not lose sight of the domain of application of that knowledge: the work of mathematics teaching
- What it looked like in practice: Sometimes preservice teachers engaged in mathematical work in the context of classroom scenarios (videos of maths lessons, samples of student work, etc.); other times they had opportunities to reflect on the implications of their (non-contextual) mathematical work for their future roles as teachers of mathematics; etc.

* Stylianides, A. J., & Stylianides, G. J. (2014). Viewing "mathematics for teaching" as a form of applied mathematics: Implications for the mathematical preparation of teachers. *Notices of the American Mathematical Society*, *61*(3), 266-276.

An intervention from the study in the area of problem solving*

* Stylianides, A. J., & Stylianides, G. J. (2014). Impacting positively on students' mathematical problem solving beliefs: An instructional intervention of short duration. *The Journal of Mathematical Behavior*, *33*, 8-29.

An intervention from the study in the area of Problem Solving (PS)

- Research context:
  - Many students of all levels of education (including preservice elementary teachers) have certain beliefs about PS that tend to influence negatively their ability or willingness to engage productively with PS (e.g., Callejo & Vila, 2009; Muis, 2004; Schoenfeld, 1998)
  - Previous interventions that achieved a positive impact on such beliefs tended to last over extended periods of time:
    - from 10 weeks (Schram et al., 1988)
    - to several years (e.g., Perrenet & Taconis, 2009; Philippou & Christou, 1998; Swars et al. 2009)
  - Prior research showed that it <u>is</u> possible to impact positively on PS beliefs but produced findings that <u>are not</u> easily usable in other contexts, including teacher education

12

The intervention lasted about 75mins and aimed to achieve 4 goals

Goal 1: To help preservice teachers recognize that problems they may perceive to be "unsolvable" can actually be solvable and within their capabilities

Corresponding counterproductive belief: "If you cannot solve a problem in a few minutes, then it's beyond your capabilities"

Goal 2: To help preservice teachers realize that effective PS requires perseverance

Goal 3: To help preservice teachers see that the formulation of mathematical problems can include more than just clearly identifiable mathematical referents (numbers or formulas)

Goal 4: To help preservice teachers appreciate that PS can be satisfying or enjoyable activity

13

A key aspect of the theoretical framework underpinning the intervention

- Individuals' counterproductive beliefs can be challenged and begin to undergo a process of change if they experience a positive episodic memory (Tulving, 1972, 1973) that is so powerful and dramatic so as to overshadow their earlier episodic memories which had shaped their current beliefs
  - How can such an episodic memory in the area of PS be engineered in the context of a mathematics class?

The intervention comprised a single mathematics problem that was purposefully designed and deliberately implemented.
From one cycle to the next we fine-tuned the design and improved our theoretical understanding of how the design supported the intended episodic memory in the context of the problem.
Over time the notion of "conceptual awareness pillars" (Stylianides & Stylianides, 2009) took a central place in the design and played a key role in its success.

14

The Blond Hair Problem*

After having many years to see each other, two friends who really loved math, Hypatia and Pythagoras, meet again. They have the following conversation:

Pythagoras: Are you married? Do you have any children? How many? How old are they?

Hypatia: Yes, I am married! I have three children and the product of their ages is 36.

Pythagoras: (After doing some thinking.) I cannot figure out their ages. I don't have enough clues.

Hypatia: Right! What if I told you that the sum of their ages is the same as the number of your address?

Pythagoras: (After doing some thinking again.) I still can't figure out their ages. I need another hint.

Hypatia: Well done! I also tell you that the oldest has blond hair.

Pythagoras: Aha! Now I can, without any doubt, figure out the ages of your children.

What are the ages of Hypatia's children? (their ages can only be natural numbers)

* Slightly modified and translated from Philippou & Christou (1995).

15

A solution to the Blond Hair Problem

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Possibility 3. Ages: 1, 3, 12	*Possibility 7.* Ages: 2, 3, 6
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18

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19

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Possibility 2. Ages: 1, 2, 18; S=21	*Possibility 6.* Ages: 2, 2, 9; S=13
Possibility 3. Ages: 1, 3, 12; S=16	*Possibility 7.* Ages: 2, 3, 6; S=11
Possibility 4. Ages: 1, 4, 9; S=14	*Possibility 8.* Ages: 3, 3, 4; S=10

20

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23

The four goals that the intervention aimed to address

Goal 1: Recognize that problems preservice teachers may perceive to be "unsolvable" can actually be solvable and within their capabilities

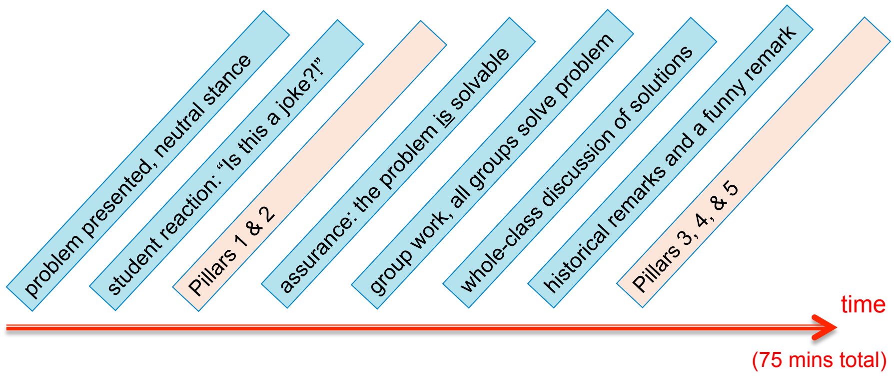
Goal 2: Realize that effective PS requires perseverance

Goal 3: See that the formulation of mathematical problems can include more than just clearly identifiable mathematical referents (numbers or formulas)

Goal 4: Appreciate that PS can be satisfying or enjoyable activity

24

Implementation plan (outline)



Pillar 1: Describe your initial reactions to the BHP.
Pillar 2: Does this problem differ in any way from most of the other problems you encountered in the mathematics classes you have taken thus far? If so, how?

(preservice teachers respond to these individually and in writing)

25

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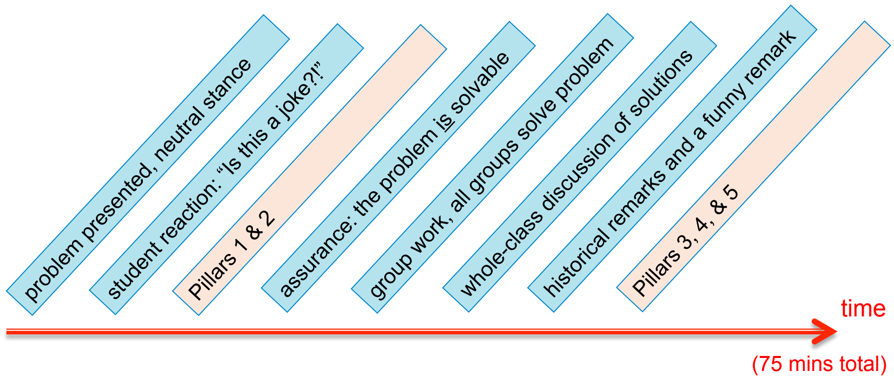

Pillar 3: Describe your experience with working on the BHP.
Pillar 4: Did the experience give you any new ideas that might be useful to you as a prospective teacher?

(preservice teachers respond to this individually and in writing)

26

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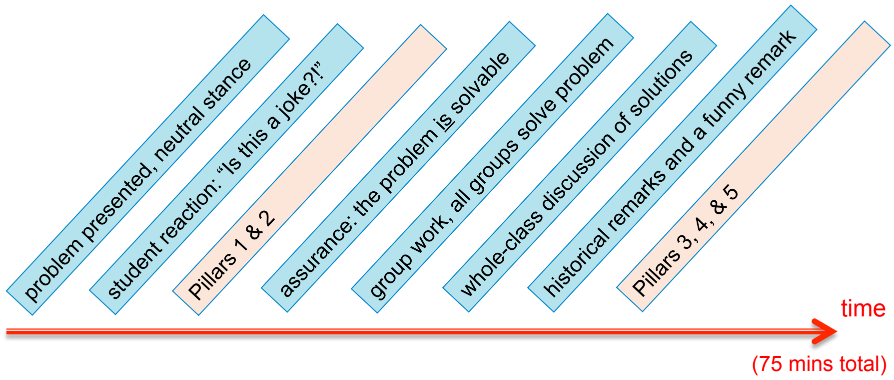


Pillar 5: (A) Read your responses and compare what you wrote at the beginning of your work on the BHP [Pillars 1 & 2] and at the end [Pillar 3].
(B) Share your observations in your small group.

(preservice teachers discuss in their small groups)

27

Data sources (total number of preservice teachers: 39)

- Data sources during the implementation of the intervention:
  - a videorecord of the implementation
  - field notes taken by a research assistant
  - participants' written responses to Pillars 1-3 (Pillar 4 useful but not directly relevant to the particular analysis)
- Relevant parts of data sources that were not specific to the intervention:
  - participants' responses to a pre- and post-course beliefs survey
  - participants' responses to a question in the final course assignment (~2 months after the intervention) asking them to identify 3 activities from the course (out of >40 activities) that contributed the most to their learning and explain why
  - individual interviews at the end of the course (~2 weeks after the final course assignment) about participants' responses to selected survey items and overall experiences in the course

28

Findings: participants' responses to the pre- and post-course beliefs survey

- Seven Likert scale items related to the goals of the intervention
- The items were phrased in a generic way
- Examples of two survey items:

> – I am afraid to make an attempt to solve a math problem that seems difficult even though it may actually be accessible to me.
> – If I cannot solve a math problem in 5-10 minutes, then I know I cannot solve it.

- Statistically significant differences between the mean values of participants' pre- and post-course responses for each of the seven survey items
  → evidence that during the course preservice teachers moved away from the four counterproductive PS beliefs targeted by the intervention
  Can this improvement in beliefs be attributed to the intervention?

29

Findings: participants' explanations in the final assignment and end-of-course interviews

- 25 preservice teachers (~65% of the total):
  - included the BHP in their list of three activities from the course that they felt contributed the most to their learning and/or
  - mentioned the BHP as an example of an activity that contributed to major changes between their pre- and post-course responses to the beliefs survey
- The explanations of these 25 preservice teachers:
  - offered evidence that the intervention helped them move away from the four counterproductive beliefs targeted by the intervention
    - Goal 1 (68%), Goal 2 (64%), Goal 3 (56%), Goal 4 (60%)
- Keep in mind:
  - These data were collected about 2 months after the intervention
  - The prompts were open-ended and non-directive
  - There were more than 40 activities from which participants could draw in their explanations

30

Erin's explanation for including the BHP in the list of 3 activities from the course with the most influence on her learning

Erin: This problem was the funniest thing we did this semester but was also significant in contributing to my learning. When we first received the problem and were asked to solve it, my whole group just sat there laughing because we thought there was no way we could solve it. Eventually […] we were able to solve it. This showed me that in math, as in life, things aren't always the way they seem. I initially gave up because the problem seemed like it simply couldn't be solved mathematically. This was important because it showed me that elementary students may also just give up if they don't immediately see connections and that it is critical to push students to examine problems more closely and look at components in ways that they are not used to looking. […]

Evidence for Goals 1, 2, & 4

- *Problems preservice teachers may perceive to be "unsolvable" can actually be solvable and within their capabilities*
- *Effective PS requires perseverance*
- *PS can be satisfying or enjoyable activity*

31

Illustrating the non-directive, open-ended nature of the interview questions

Background context*: Andria said that the course helped her appreciate that problem solving can be "fun." In the excerpt below the interviewer followed up on Andria's comment.*

- *Interviewer*: I would be interested to understand more how you think about fun problems. Are there any examples from the course that you thought were fun?
- *Andria*: I don't know why because it was really frustrating but I found the Blond Hair Problem to be really fun and it was the problem where I looked at it at first and I was like, "This is a joke," and once we realized it wasn't a joke it was cool to work through it and figure it out. […] With the Blond Hair Problem I had no idea what to expect because I had never seen a problem like that before. For some reason that problem was fun to me and I don't know why because it was frustrating….

Evidence for Goal 4:

- PS can be satisfying or enjoyable activity

32

Illustrating our analysis of participants' responses to the Pillars

Beginning of intervention
Pillar 1: Describe your initial reactions to the BHP.
Pillar 2: Does this problem differ in any way from most of the other problems you encountered in the mathematics classes you have taken thus far? If so, how?
End of intervention
Pillar 3: Describe your experience with working on the BHP.

Goal 1: Recognize that problems preservice teachers may perceive to be "unsolvable" can actually be solvable and within their capabilities

Laney's response to Pillar 1: I think it's impossible to figure out. We don't know Pythagora's address so we don't know the sum, and there are too many possible answers to the product of 36. […] Also, the part about the oldest being blonde seems very irrelevant and doesn't help at all.

Laney's response to Pillar 3: It made me realize that my initial thoughts about the problem were completely wrong and this problem was possible. I now understand that problems that seem impossible or seem to have irrelevant parts might actually be able to be solved. Before dismissing any problem, put some real effort into it and think about it in numerous ways.

33

Findings: participants' responses to the Pillars

Goals of the intervention	% of participants whose responses to the pillars offered evidence for each goal (calculated using N=39, *actual N should be lower*)
1. Recognize that problems preservice teachers may perceive to be "unsolvable" can actually be solvable and within their capabilities	54
2. Realize that effective PS requires perseverance	77
3. See that the formulation of mathematical problems can include more than just clearly identifiable mathematical referents (numbers or formulas)	36
4. Appreciate that effective PS can be satisfying or enjoyable activity	36

34

Concluding remarks

- Many elementary teachers start teaching with beliefs about PS that we know are unlikely to underpin productive mathematics instruction
- Prior research offered few solutions to this problem of teachers' subject-related beliefs but these solutions are not easily usable due to their long duration
- This study took a step toward addressing the issue of whether a "quicker solution" might also be possible
  - It offered evidence for the effectiveness of an intervention of *short duration*
  - It developed a theoretical framework explaining how the intervention 'works' thus supporting its use/adaptation in other contexts
  - Would the intervention have a similar effect in a course that did not place a high premium on PS and reasoning? (future research)

35

Another example of an intervention of short duration from the study

36

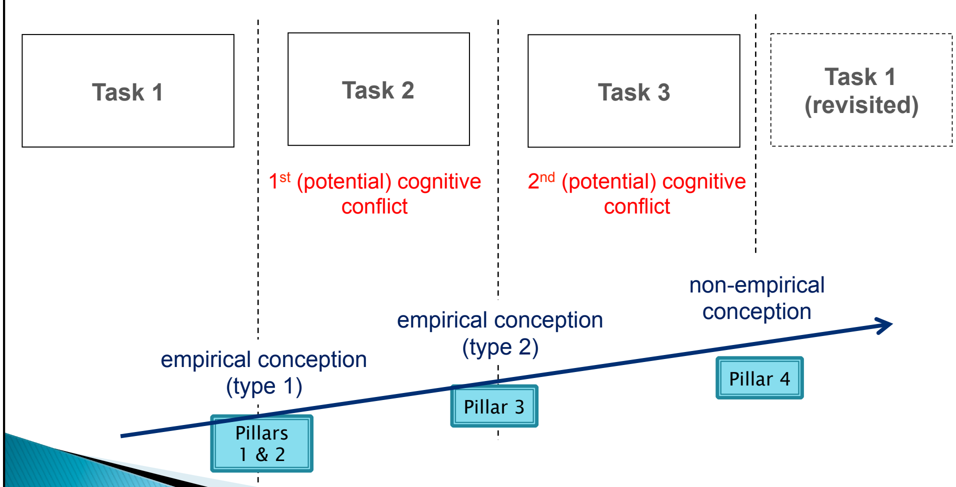
An intervention in the area of proof*

- Research context:
  - A large body of research showed that many students of all levels of education including preservice elementary teachers have the misconception that empirical arguments are proofs of mathematical generalizations (e.g., Harel & Sowder, 2007; Morris, 2002, 2007)
  - Some even concluded that misconceptions such as this one are resistant to remediation in the context of a 1yr elementary teacher education program (Goulding & Suggate, 2001)
- The intervention:
  - Duration: <3hrs
  - Key theoretical constructs:
    - "cognitive conflict" (e.g., Piaget, 1985)
    - "example spaces" (Watson & Mason, 2005)
    - "pivotal counterexamples" (Zazkis & Chernoff, 2008)
    - "conceptual awareness pillars" (Stylianides & Stylianides, 2009)

* Stylianides, G. J., & Stylianides, A. J. (2009). Facilitating the transition from empirical arguments to proof. *Journal for Research in Mathematics Education*, *40*, 314-352.

37

Outline of the intervention (<3hrs)



Can we design interventions of short duration in teacher education to help alleviate important and persistent problems of preservice teachers' subject knowledge (including subject-related beliefs)?

The two interventions offer suggestive evidence that it is feasible to design interventions of short duration in mathematics teacher education to alleviate significant problems of preservice elementary teachers' mathematics subject knowledge.

More research is needed on the design of such interventions and on what might be involved in their (adaptation for) use in different teacher education programs.

This research is timely in light of:

- *the current pressures on teacher education programs to cover too much in too little time*
- *the growing need to elevate the research/theoretical basis of provision in teacher education programs*

Intervention-based research in (mathematics) teacher education*

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41