Maynooth University
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Using the STACK question type in Moodle.

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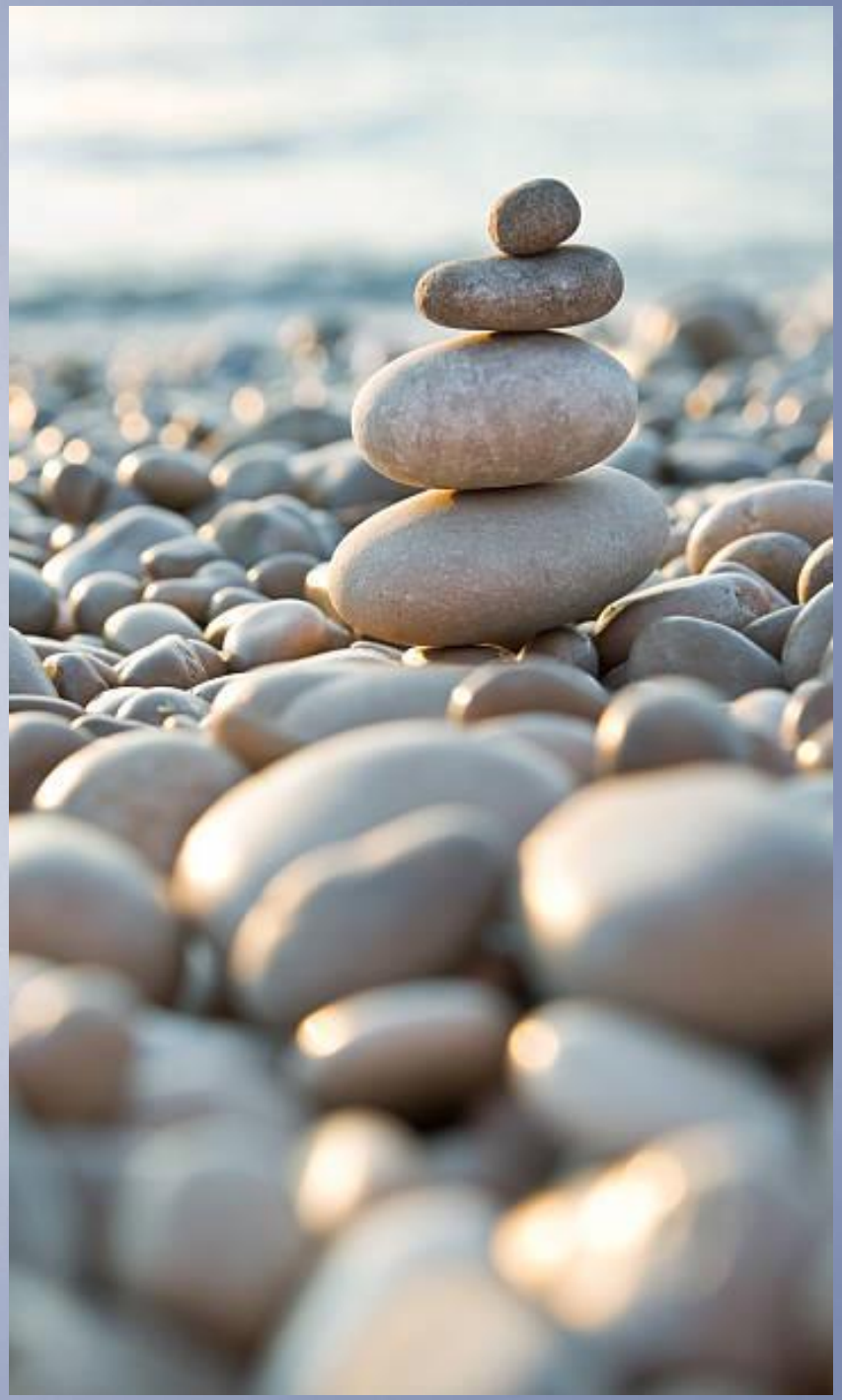


Context

Limited use of automated assessments pre-pandemic, predominantly paper based assessments.

Switch to online quizzes and PDF uploads for continuous assessments and final exams during the pandemic.

Available question types were limited.
Prompted testing of other options.

Returning to in person paper based final exams.
Quizzes still used as part of continuous assessments.

Context

Mixture of formative & summative quiz assessments

Added JSXGraph based questions

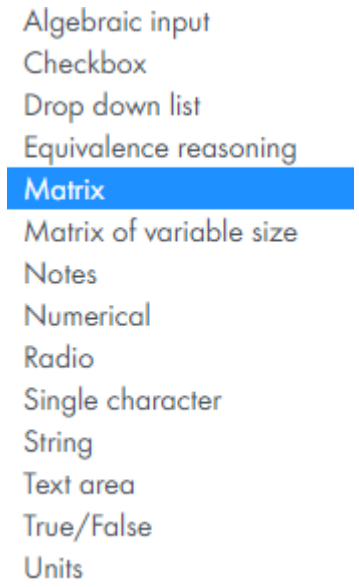
Work with lecturing staff to produce questions.

Why use the STACK question type?

Flexibility

- Multiple input types
  - Algebraic expressions
  - Matrices
  - Numerical
  - Multi-choice

  Full list of input types:



- Input validation for users
- Randomisation
- Complex marking
- JSXgraph support

Input types

Numerical & Algebraic input

Numerical

Consider the points $x = (0, -2, 0, 1, 1), y = (-3, 1, 2, -1, -3)$ in $\mathbb{R}^5$.

Write down the cosine of the angle between x and y.

$\cos(\theta) =$ -(1/2) $-\frac{1}{2}$

Let $f(x) = \frac{x}{6\text{-}x^2+7}$. Then

f has a local minimum at $x =$ -(sqrt(7)/sqrt(6))

Algebraic input

Find the first order partial derivatives of:

$$f(x, y) = -3 \cdot \cos(y) + 13 \cdot x \cdot y^{13} - 23 \cdot x^2$$

$f_x =$ 13*y^13-46*x

$$f(x) = (x^3 - 3) \cdot e^{6 \cdot x}$$

The second derivative is

$f''(x) =$ (16*x^6+48*x^5+30*x^4-80)*e^(4*x)

Your last answer was interpreted as follows:

$$(16 \cdot x^6 + 48 \cdot x^5 + 30 \cdot x^4 - 80) \cdot e^{4 \cdot x}$$

The variables found in your answer were: $[x]$

Input types

Matrices – fixed & variable sized

Consider the system of linear equations:

$$\begin{array}{rrrrcl} -3 \cdot x_1 & +6 \cdot x_2 & +3 \cdot x_3 & -x_4 & = & 0 \\ x_1 & -2 \cdot x_2 & +x_3 & & = & 2 \\ & x_2 & & & = & 3 \\ & -2 \cdot x_2 & +x_3 & & = & 3 \end{array}$$

(a) Write down the augmented matrix of the system of linear equations.

$$\begin{bmatrix} -3 & 6 & 3 & -1 & 0 \\ 1 & -2 & 1 & 0 & 2 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & -2 & 1 & 0 & 3 \end{bmatrix}$$

Given the matrices

$$A = \begin{bmatrix} 4 & 4 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & -1 \\ 5 & 3 \\ 4 & 4 \end{bmatrix}, \quad C = \begin{bmatrix} 5 & -2 & 1 \end{bmatrix}$$

(a) Compute the matrix $2A + 4I$ where I is the 2×2 identity

$2A + 4I =$ [12 8
2 6]

Your last answer was interpreted as follows:

$$\begin{bmatrix} 12 & 8 \\ 2 & 6 \end{bmatrix}$$

Input types

❑ Dropdown

(1) $\sum_{n=0}^{\infty} \frac{5^{4\cdot n-1}}{8^{5\cdot n+5}}$

Determine $\lim_{n\to\infty} |x_n|^{\frac{1}{n}} =$

Henc

✓ (No answer given)
the series converges
the series does not converge
the root test is inconclusive

If you

(2) $\sum_{n=1}^{\infty} \left(\frac{25\cdot n+1}{9\cdot n-3}\right)^{n/2}$

Determine $\lim_{n\to\infty} |x_n|^{\frac{1}{n}} =$

Hence (No answer given)

If you can state the limit L of the series. $L =$

• Radio buttons

Find the Taylor series of $f(x) = \frac{1}{1-3\cdot x}$ at $c = 0$.

- $\sum_{n=0}^{\infty} 3^n \cdot x^n$
- $\sum_{n=0}^{\infty} \left(\frac{x}{3}\right)^n$
- $\sum_{n=0}^{\infty} \frac{1}{n!} \cdot \left(\frac{x}{3}\right)^n$
- $\sum_{n=0}^{\infty} \frac{3^n}{n!} \cdot x^n$
- None of these

✔ Correct answer, well done.

✓ Checkbox

Consider the function

$$f(x) = \begin{cases} 2\cdot x+1 & x \le -2 \\ 2\cdot (x+1)^3 & -2 < x < 1 \\ 4\cdot (x+1)^2 & x \ge 1 \end{cases},$$

Tick all of the following statements that are correct.

- ☑ $\lim_{x\to -2^+} f(x) = -2.$
- ☐ $\lim_{x\to 2} f(x) = -3.$
- ☐ $\lim_{x\to -2} f(x)$ exists.
- ☑ $\lim_{x\to 1^-} f(x) = 16.$
- ☑ $\lim_{x\to 0} f(x) = 2.$
- ☐ $\lim_{x\to -2^-} f(x) = \lim_{x\to -2^+} f(x).$
- ☐ $\lim_{x\to -2^-} f(x) = -16.$
- ☑ $\lim_{x\to 1} f(x)$ exists.

Multi-choice

Input Validation

General

$\frac{\partial L}{\partial x} =$ 2*x+lambda

Your last answer was interpreted as follows:

$2 \cdot x + \lambda$

The variables found in your answer were: $[x, \lambda]$

Input Validation

Matrices

(a) Write down the augmented matrix of the system of linear equations.

3	9	-3	-7	-1
1	3	1	-2	1
0	1	0	0	-1
	-3	1	0	

Your last answer was interpreted as follows:

$$\begin{bmatrix} 3 & 9 & -3 & -7 & -1 \\ 1 & 3 & 1 & -2 & 1 \\ 0 & 1 & 0 & 0 & -1 \\ ? & -3 & 1 & 0 & ? \end{bmatrix}$$

This answer is invalid. Your answer contains question mark characters, ?, which are not permitted in answers. You should replace these with a specific value.

Input Validation

Numerical

Suppose 6.0% of a population is infected with a certain disease. On a medical test 95% of those infected give a positive result, while 2% of those not infected give a positive result. For a randomly chosen person from this population:

(a) What is the probability that he/she does not have the disease?

p=47/50

Your last answer was interpreted as follows:

$$p = \frac{47}{50}$$

This answer is invalid. This input expects a number, and so may not contain variables.

The variables found in your answer were: $[p]$

(b) What is the probability that he/she tests positive?

x^2

Your last answer was interpreted as follows:

$$x^2$$

This answer is invalid. This input expects a number, and so may not contain variables.

The variables found in your answer were: $[x]$

Randomisation

$$f(x) = \left(x^{6} - 6\right) \cdot e^{6 \cdot x}$$

The second derivative is

$f''(x) =$

$$f(x) = \left(x^3 - 1\right) \cdot e^{4 \cdot x}$$

The second derivative is $f''(x) = \left(16 \cdot x^3 + 24 \cdot x^2 + 6 \cdot x - 16\right) \cdot e^{4 \cdot x}$

$$f(x) = \left(x^3 - 3\right) \cdot e^{6 \cdot x}$$

The second derivative is $f''(x) = \left(36 \cdot x^3 + 36 \cdot x^2 + 6 \cdot x - 108\right) \cdot e^{6 \cdot x}$

$$f(x) = \left(x^3 - 7\right) \cdot e^{3 \cdot x}$$

The second derivative is $f''(x) = \left(9 \cdot x^3 + 18 \cdot x^2 + 6 \cdot x - 63\right) \cdot e^{3 \cdot x}$

Deployed Variants

$$f(x) = \left(x^6 - 6\right) \cdot e^{6 \cdot x}$$

The second derivative is

$f''(x) =$

Variant	Question note
2006574530	$f(x) = \left(x^3 - 1\right) \cdot e^{4 \cdot x}$ The second derivative is $f''(x) = \left(16 \cdot x^3 + 24 \cdot x^2 + 6 \cdot x - 16\right) \cdot e^{4 \cdot x}$
1759236276	$f(x) = \left(x^3 - 3\right) \cdot e^{6 \cdot x}$ The second derivative is $f''(x) = \left(36 \cdot x^3 + 36 \cdot x^2 + 6 \cdot x - 108\right) \cdot e^{6 \cdot x}$
953820835	$f(x) = \left(x^3 - 7\right) \cdot e^{3 \cdot x}$ The second derivative is $f''(x) = \left(9 \cdot x^3 + 18 \cdot x^2 + 6 \cdot x - 63\right) \cdot e^{3 \cdot x}$

Main uses:

- Prevent potential errors
- In large scale assessments it improves performance and reduces system load.

Randomisation

If \(X\) is {@situation[2]@}, then an appropriate probability distribution for \(X\) is the

[[input:ans1]] [[validation:ans1]]

If X is the total number of heads obtained when tossing a coin 5 times, then an appropriate probability distribution for X is the

- Poisson distribution
- Binomial distribution
- Uniform distribution
- Bernoulli distribution
- None of the above

If X is the number of earthquakes that happen in a year in a particular country, then an appropriate probability distribution for X is the

- Binomial distribution
- Bernoulli distribution
- Poisson distribution
- Uniform distribution
- None of the above

Complex marking

Using the students answer

Let

$$f(x) = \frac{4}{5 \cdot x^2 - 19}.$$

[2 marks] (a) Evaluate $f(-2) =$ 4 .

Your last answer was interpreted as follows:

4

✔ Correct answer, well done.

Marks for this submission: 0.13/0.13.

[3 marks] (b) What is the slope of the tangent line to $y = f(x)$ at $x = -2$? -80

Your last answer was interpreted as follows:

−80

◑ Your answer is partially correct.

Marks for this submission: 0.13/0.20.

[4 marks] (c) The equation of the tangent line to the curve $y = f(x)$ at $x = -2$ is of the form $y = ax + b$ whe

$a =$ -80

Your last answer was interpreted as follows:

−80

✔ Correct answer, well done.

Marks for this submission: 0.13/0.13.

and $b =$ -156

Your last answer was interpreted as follows:

2* ○ -- −156

✔ Correct answer, well done.

Marks for this submission: 0.13/0.13.

JSXGraph + STACK

- JSXGraph is a javascript plotting library supporting interactive graphics.
- Allows for visual aids or demonstrations of concepts visually.
- STACK supports inclusion of JSXGraph in a special JSXGraph code block.
- STACK also supports grading variables passed from JSXGraph to the CAS.

Randomized graphics

Which of the following sets is shown in red on the numberline below?

Note: you can zoom in and out with the + and − buttons, the o button resets the zoom, and you can move the view with the arrows buttons. You can also hover over the points to see their coordinates.

- o + ← ↓ ↑ →

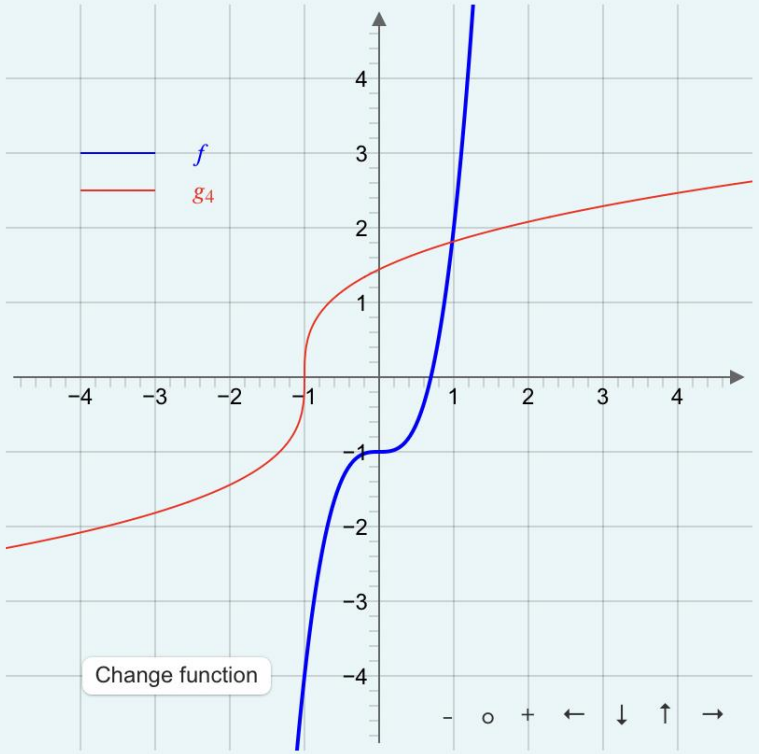
- (No answer given)
- $\{x \in \mathbb{R} : 3 \leq 3 - x\} \cup \{x \in \mathbb{R} : 8 \leq 2 \cdot x < 16\}.$
- $\{x \in \mathbb{R} : 3 \leq 3 - x\} \cup \{x \in \mathbb{R} : 8 \leq 2 \cdot x \leq 16\}.$
- $\{x \in \mathbb{R} : 3 \leq 3 - x\} \cup \{x \in \mathbb{R} : 8 < 2 \cdot x \leq 16\}.$
- None of these

Graphics as an aid

The function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = 3 \cdot x^3 - 1$$

is plotted in the figure below in blue.



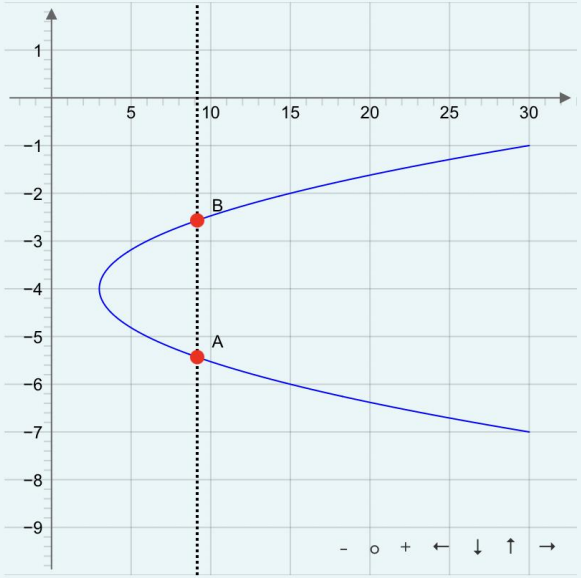
Which one of the following functions, $g_1(x), g_2(x), g_3(x), g_4(x)$ defined below, is the inverse function $f^{-1}(x)$ of $f(x)$. You can press the 'change function' button to show the plots of the functions $g_1(x), g_2(x), g_3(x), g_4(x)$ in red.

- (No answer given)
- $g_1(x) = \frac{(x+1)^3}{27}$
- $g_2(x) = \frac{(x+1)^{\frac{1}{3}}}{3^{\frac{1}{3}}}$
- $g_3(x) = 3^{\frac{1}{3}} \cdot (x+1)^{\frac{1}{3}}$
- $g_4(x) = \frac{(x-1)^{\frac{1}{3}}}{3^{\frac{1}{3}}}$

Interactive visuals

Decide by using the vertical line test, whether the blue curve shown below is the graph of a function, that is, it can be written in the form $y = f(x)$.

Note: You can click and drag the dashed black vertical line to help you apply the vertical line test.

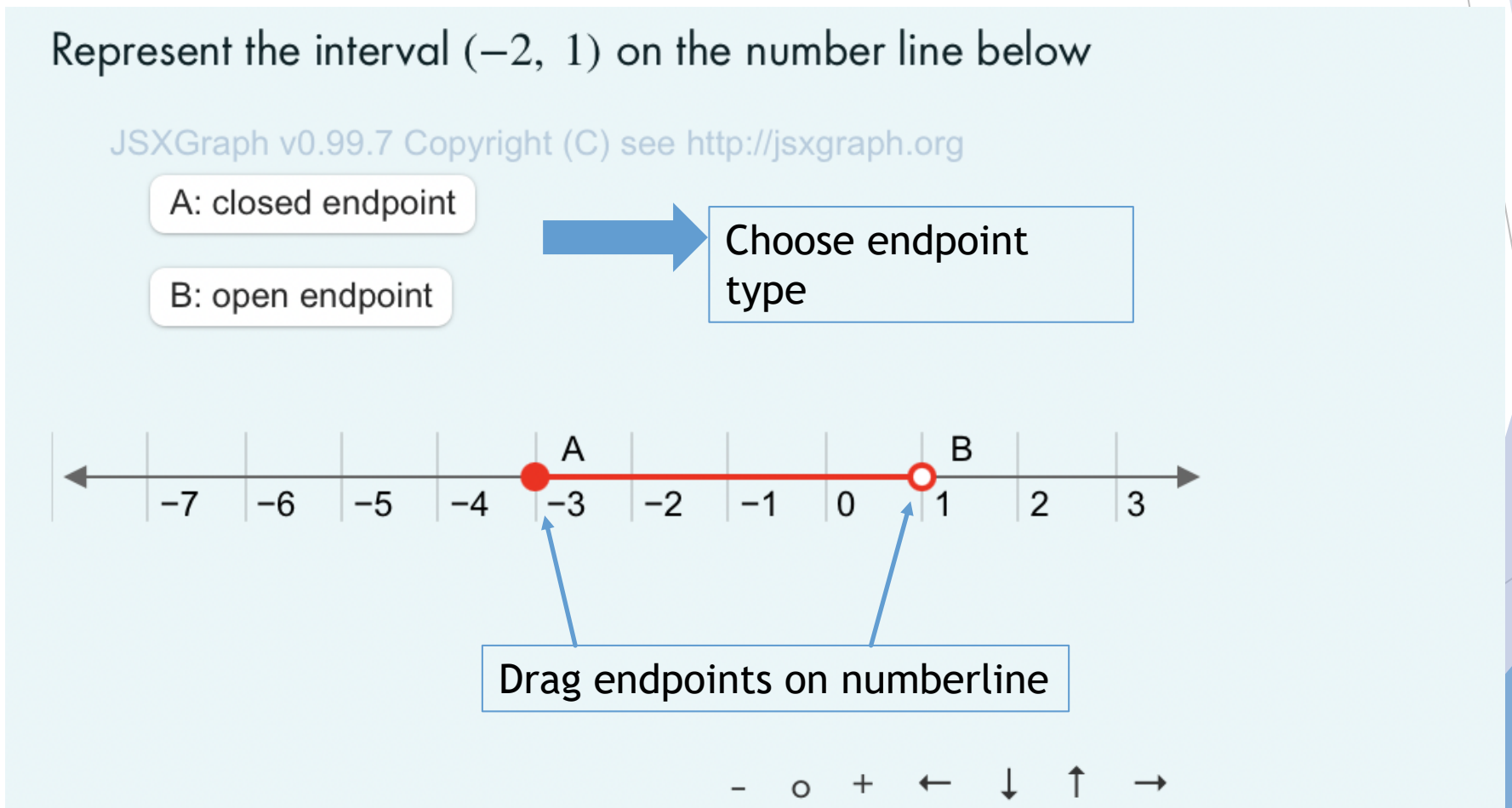


The graph shown above is the graph of a function according to the vertical line test:

- (No answer given)
- Yes
- No

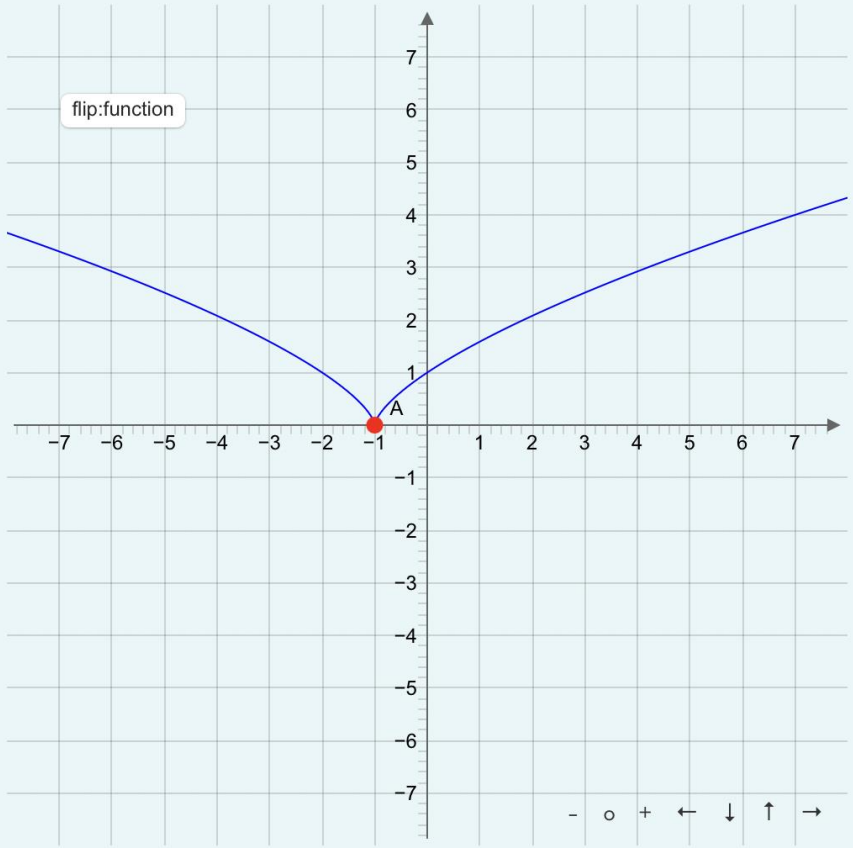
Graph as an answer

Represent the interval $(-2,\ 1)$ on the number line below



Graph as an answer

Consider the graph of $y = f(x)$ below, where $f(x) = (x+1)^{\frac{2}{3}}$.

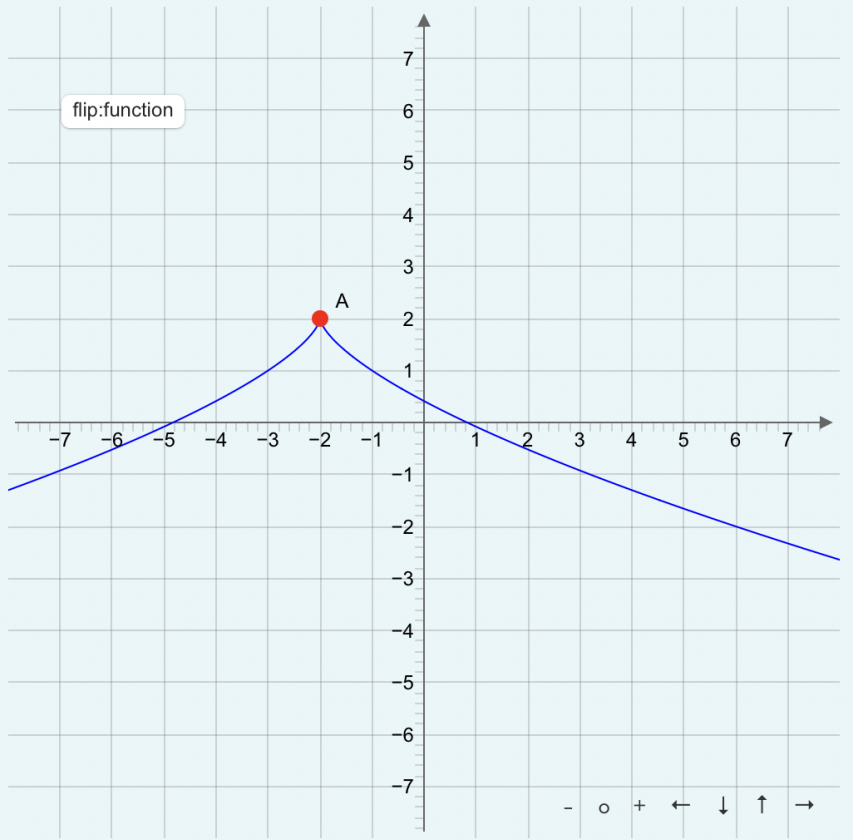


By transforming the graph of f, sketch the graph of $y = f(x+1)$.

Interactive

- Drag red point
- Flip function

Consider the graph of $y = f(x)$ below, where $f(x) = (x+1)^{\frac{2}{3}}$.



By transforming the graph of f, sketch the graph of $y = f(x+1)$.

In summary, the strength of STACK is its flexibility.

STACK's use of a computer algebra system (based on Maxima) provides a powerful tool for creating questions and for assessing student responses.

Randomisation: Numbers, list elements, text.

Complex marking: Multiple nodes, can use incorrect answer to give partial marks in later parts of a question.

Input Validation: Students can check their answers are interpreted as expected.

Feedback options: Can be particularly helpful in formative quizzes.
We hope to dedicate more time to this in the future.

Note:
Knowledge of LaTeX and Maxima is required to fully utilise STACK.
Javascript knowledge is needed for JSXgraph.